Mathematica 11.3 Integration Test Results

Test results for the 115 problems in "1.1.2.5 (a+b x^2)^p (c+d x^2)^q (e+f x^2)^r.m"

Problem 23: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(a+b x^2\right) \left(c+d x^2\right)^{3/2} \sqrt{e+f x^2}\, dx$$

Optimal (type 4, 544 leaves, 7 steps):

$$-\left(\left(\left(7 a d f \left(2 d^2 e^2-7 c d e f-3 c^2 f^2\right)-b \left(8 d^3 e^3-19 c d^2 e^2 f+9 c^2 d e f^2-6 c^3 f^3\right)\right) x \sqrt{c+d x^2}\right)\Big/\left(105 d^2 f^2 \sqrt{e+f x^2}\right)\right)+\frac{1}{105 d f^2}\left(7 a d f (d e+3 c f)-b \left(4 d^2 e^2-6 c d e f+6 c^2 f^2\right)\right) x \sqrt{c+d x^2} \sqrt{e+f x^2}+\frac{(b d e-2 b c f+7 a d f)\, x \left(c+d x^2\right)^{3/2} \sqrt{e+f x^2}}{35 d f}+\frac{b x \left(c+d x^2\right)^{5/2} \sqrt{e+f x^2}}{7 d}+\left(\sqrt{e} \left(7 a d f \left(2 d^2 e^2-7 c d e f-3 c^2 f^2\right)-b \left(8 d^3 e^3-19 c d^2 e^2 f+9 c^2 d e f^2-6 c^3 f^3\right)\right) \sqrt{c+d x^2}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]\right)\Bigg/\left(105 d^2 f^{5/2} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right)-\left(e^{3/2} \left(7 a d f (d e-9 c f)-b \left(4 d^2 e^2-9 c d e f-3 c^2 f^2\right)\right) \sqrt{c+d x^2}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]\right)\Bigg/\left(105 d f^{5/2} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right)$$

Result (type 4, 373 leaves):

$$\frac{1}{105\, d \sqrt{\frac{d}{c}}\, f^3 \sqrt{c+d x^2}\, \sqrt{e+f x^2}} \left(\sqrt{\frac{d}{c}}\, f x \left(c+d x^2\right) \left(e+f x^2\right) \left(7 a d f \left(6 c f+d \left(e+3 f x^2\right)\right)+b \left(3 c^2 f^2+3 c d f \left(3 e+8 f x^2\right)+d^2 \left(-4 e^2+3 e f x^2+15 f^2 x^4\right)\right)\right)+ i\, e \left(7 a d f \left(2 d^2 e^2-7 c d e f-3 c^2 f^2\right)+b \left(-8 d^3 e^3+19 c d^2 e^2 f-9 c^2 d e f^2+6 c^3 f^3\right)\right) \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right], \frac{c f}{d e}\right] - i\, e \left(-d e+c f\right) \left(-14 a d f \left(d e-3 c f\right)+b \left(8 d^2 e^2-15 c d e f+3 c^2 f^2\right)\right) \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right], \frac{c f}{d e}\right]\right)$$

Problem 24: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(a+b x^2\right) \sqrt{c+d x^2}\, \sqrt{e+f x^2}\, dx$$

Optimal (type 4, 381 leaves, 6 steps):

$$\frac{\left(5 a d f \left(d e+c f\right)-2 b \left(d^2 e^2-c d e f+c^2 f^2\right)\right) x \sqrt{c+d x^2}}{15 d^2 f \sqrt{e+f x^2}}+\frac{\left(b d e-2 b c f+5 a d f\right) x \sqrt{c+d x^2}\, \sqrt{e+f x^2}}{15 d f}+\frac{b x \left(c+d x^2\right)^{3/2} \sqrt{e+f x^2}}{5 d}-\left(\sqrt{e} \left(5 a d f \left(d e+c f\right)-2 b \left(d^2 e^2-c d e f+c^2 f^2\right)\right) \sqrt{c+d x^2}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]\right) \Big/ \left(15 d^2 f^{3/2} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}}\, \sqrt{e+f x^2}\right)-\left(e^{3/2} \left(b d e+b c f-10 a d f\right) \sqrt{c+d x^2}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]\right) \Big/ \left(15 d f^{3/2} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}}\, \sqrt{e+f x^2}\right)$$

Result (type 4, 267 leaves):

$$\left(\sqrt{\frac{d}{c}}\, f\, x\, \left(c+d\, x^2\right)\, \left(e+f\, x^2\right)\, \left(b\, c\, f+5\, a\, d\, f+b\, d\, \left(e+3\, f\, x^2\right)\right)+\right.$$

$$i\, e\, \left(-5\, a\, d\, f\, \left(d\, e+c\, f\right)+2\, b\, \left(d^2\, e^2-c\, d\, e\, f+c^2\, f^2\right)\right)\, \sqrt{1+\frac{d\, x^2}{c}}\, \sqrt{1+\frac{f\, x^2}{e}}$$

$$\text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right],\, \frac{c\, f}{d\, e}\right]-i\, e\, \left(-d\, e+c\, f\right)\, \left(-2\, b\, d\, e+b\, c\, f+5\, a\, d\, f\right)\, \sqrt{1+\frac{d\, x^2}{c}}$$

$$\left.\sqrt{1+\frac{f\, x^2}{e}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right],\, \frac{c\, f}{d\, e}\right]\right)\Big/\left(15\, d\, \sqrt{\frac{d}{c}}\, f^2\, \sqrt{c+d\, x^2}\, \sqrt{e+f\, x^2}\right)$$

Problem 25: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b\, x^2\right)\, \sqrt{e+f\, x^2}}{\sqrt{c+d\, x^2}}\, dx$$

Optimal (type 4, 283 leaves, 5 steps):

$$\frac{\left(b\, d\, e-2\, b\, c\, f+3\, a\, d\, f\right)\, x\, \sqrt{c+d\, x^2}}{3\, d^2\, \sqrt{e+f\, x^2}}+\frac{b\, x\, \sqrt{c+d\, x^2}\, \sqrt{e+f\, x^2}}{3\, d}-$$

$$\left(\sqrt{e}\, \left(b\, d\, e-2\, b\, c\, f+3\, a\, d\, f\right)\, \sqrt{c+d\, x^2}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right],\, 1-\frac{d\, e}{c\, f}\right]\right)\Big/$$

$$\left(3\, d^2\, \sqrt{f}\, \sqrt{\frac{e\, \left(c+d\, x^2\right)}{c\, \left(e+f\, x^2\right)}}\, \sqrt{e+f\, x^2}\right)-$$

$$\frac{\left(b\, c-3\, a\, d\right)\, e^{3/2}\, \sqrt{c+d\, x^2}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right],\, 1-\frac{d\, e}{c\, f}\right]}{3\, c\, d\, \sqrt{f}\, \sqrt{\frac{e\, \left(c+d\, x^2\right)}{c\, \left(e+f\, x^2\right)}}\, \sqrt{e+f\, x^2}}$$

Result (type 4, 212 leaves):

$$\left(b\sqrt{\frac{d}{c}}\,f\,x\,\left(c+d\,x^2\right)\,\left(e+f\,x^2\right)+\right.$$
$$\mathrm{i}\,e\,\left(-b\,d\,e+2\,b\,c\,f-3\,a\,d\,f\right)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]-$$
$$\left.\mathrm{i}\,b\,e\,\left(-d\,e+c\,f\right)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]\right)\Bigg/$$
$$\left(3\,d\,\sqrt{\frac{d}{c}}\,f\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}\right)$$

Problem 26: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x^2\right)\,\sqrt{e+f\,x^2}}{\left(c+d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 271 leaves, 5 steps):

$$\frac{\left(2\,b\,c-a\,d\right)\,f\,x\,\sqrt{c+d\,x^2}}{c\,d^2\,\sqrt{e+f\,x^2}}-\frac{\left(b\,c-a\,d\right)\,x\,\sqrt{e+f\,x^2}}{c\,d\,\sqrt{c+d\,x^2}}-$$
$$\frac{\left(2\,b\,c-a\,d\right)\,\sqrt{e}\,\sqrt{f}\,\sqrt{c+d\,x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],\,1-\frac{d\,e}{c\,f}\right]}{c\,d^2\,\sqrt{\frac{e\,\left(c+d\,x^2\right)}{c\,\left(e+f\,x^2\right)}}\,\sqrt{e+f\,x^2}}+$$
$$\frac{b\,e^{3/2}\,\sqrt{c+d\,x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],\,1-\frac{d\,e}{c\,f}\right]}{c\,d\,\sqrt{f}\,\sqrt{\frac{e\,\left(c+d\,x^2\right)}{c\,\left(e+f\,x^2\right)}}\,\sqrt{e+f\,x^2}}$$

Result (type 4, 192 leaves):

$$\left(-\mathrm{i}\,\left(2\,b\,c-a\,d\right)\,e\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]-\right.$$
$$\left.\left(b\,c-a\,d\right)\,\left(\sqrt{\frac{d}{c}}\,x\,\left(e+f\,x^2\right)-\mathrm{i}\,e\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]\right)\right)\Bigg/$$
$$\left(c^2\,\left(\frac{d}{c}\right)^{3/2}\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}\right)$$

Problem 27: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^2\right) \sqrt{e+f x^2}}{\left(c+d x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 274 leaves, 4 steps):

$$-\frac{(b c-a d) x \sqrt{e+f x^2}}{3 c d \left(c+d x^2\right)^{3/2}}+\left((d (b c+2 a d) e-c (2 b c+a d) f) \sqrt{e+f x^2} \, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1-\frac{c f}{d e}\right]\right) \Bigg/ \left(3 c^{3/2} d^{3/2} (d e-c f) \sqrt{c+d x^2} \sqrt{\frac{c \left(e+f x^2\right)}{e \left(c+d x^2\right)}}\right)+\frac{(b c-a d) e^{3/2} \sqrt{f} \sqrt{c+d x^2} \, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]}{3 c^2 d (d e-c f) \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}}$$

Result (type 4, 297 leaves):

$$\frac{1}{3 c^3 \left(\frac{d}{c}\right)^{3/2} (-d e+c f) \left(c+d x^2\right)^{3/2} \sqrt{e+f x^2}} \left(\sqrt{\frac{d}{c}} x \left(e+f x^2\right) \left(a d \left(-3 c d e+2 c^2 f-2 d^2 e x^2+c d f x^2\right)+b c \left(c^2 f-d^2 e x^2+2 c d f x^2\right)\right)+ i e (a d (-2 d e+c f)+b c (-d e+2 c f)) \left(c+d x^2\right) \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \, \text{EllipticE}\left[i \, \text{ArcSinh}\left[\sqrt{\frac{d}{c}} x\right], \frac{c f}{d e}\right]- i (b c+2 a d) e (-d e+c f) \left(c+d x^2\right) \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \, \text{EllipticF}\left[i \, \text{ArcSinh}\left[\sqrt{\frac{d}{c}} x\right], \frac{c f}{d e}\right]\right)$$

Problem 28: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^2\right) \sqrt{e+f x^2}}{\left(c+d x^2\right)^{7/2}} \, dx$$

Optimal (type 4, 385 leaves, 5 steps):

$$-\frac{(b c-a d)\, x \sqrt{e+f x^2}}{5 c d \left(c+d x^2\right)^{5/2}}+\frac{(a d (4 d e-3 c f)+b c (d e-2 c f))\, x \sqrt{e+f x^2}}{15 c^2 d (d e-c f) \left(c+d x^2\right)^{3/2}}+$$
$$\left(\left(2 b c \left(d^2 e^2-c d e f+c^2 f^2\right)+a d \left(8 d^2 e^2-13 c d e f+3 c^2 f^2\right)\right) \sqrt{e+f x^2}\right.$$
$$\left.\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], 1-\frac{c f}{d e}\right]\right) \Big/ \left(15 c^{5/2} d^{3/2} (d e-c f)^2 \sqrt{c+d x^2} \sqrt{\frac{c \left(e+f x^2\right)}{e \left(c+d x^2\right)}}\right)-$$
$$\left(e^{3/2} \sqrt{f}\, (2 a d (2 d e-3 c f)+b c (d e+c f)) \sqrt{c+d x^2}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]\right) \Big/$$
$$\left(15 c^3 d (d e-c f)^2 \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right)$$

Result (type 4, 379 leaves):

$$\frac{1}{15 c^4 \left(\frac{d}{c}\right)^{3/2} (d e-c f)^2 \left(c+d x^2\right)^{5/2} \sqrt{e+f x^2}}$$
$$\left(-\sqrt{\frac{d}{c}}\, x \left(e+f x^2\right) \left(3 c^2 (b c-a d) (d e-c f)^2-c (d e-c f) (a d (4 d e-3 c f)+b c (d e-2 c f))\right.\right.$$
$$\left.\left(c+d x^2\right)-\left(2 b c \left(d^2 e^2-c d e f+c^2 f^2\right)+a d \left(8 d^2 e^2-13 c d e f+3 c^2 f^2\right)\right) \left(c+d x^2\right)^2\right)+$$
$$i\, e \left(c+d x^2\right)^2 \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \left(\left(2 b c \left(d^2 e^2-c d e f+c^2 f^2\right)+\right.\right.$$
$$\left.a d \left(8 d^2 e^2-13 c d e f+3 c^2 f^2\right)\right) \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right], \frac{c f}{d e}\right]-$$
$$\left.\left.(-d e+c f) (b c (-2 d e+c f)+a d (-8 d e+9 c f))\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right], \frac{c f}{d e}\right]\right)\right)$$

Problem 29: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(a+b x^2\right) \sqrt{c+d x^2} \left(e+f x^2\right)^{3/2} dx$$

Optimal (type 4, 543 leaves, 7 steps):

$$\left(\left(7\,a\,d\,f\,\left(3\,d^2\,e^2+7\,c\,d\,e\,f-2\,c^2\,f^2\right)-b\,\left(6\,d^3\,e^3-9\,c\,d^2\,e^2\,f+19\,c^2\,d\,e\,f^2-8\,c^3\,f^3\right)\right)\,x\,\sqrt{c+d\,x^2}\right)\Big/\left(105\,d^3\,f\,\sqrt{e+f\,x^2}\right)+\frac{1}{105\,d^2\,f}\left(14\,a\,d\,f\,\left(3\,d\,e-c\,f\right)+b\,\left(3\,d^2\,e^2-15\,c\,d\,e\,f+8\,c^2\,f^2\right)\right)\,x\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}+$$

$$\frac{\left(3\,b\,d\,e-4\,b\,c\,f+7\,a\,d\,f\right)\,x\,\left(c+d\,x^2\right)^{3/2}\,\sqrt{e+f\,x^2}}{35\,d^2}+\frac{b\,x\,\left(c+d\,x^2\right)^{3/2}\,\left(e+f\,x^2\right)^{3/2}}{7\,d}-$$

$$\left(\sqrt{e}\,\left(7\,a\,d\,f\,\left(3\,d^2\,e^2+7\,c\,d\,e\,f-2\,c^2\,f^2\right)-b\,\left(6\,d^3\,e^3-9\,c\,d^2\,e^2\,f+19\,c^2\,d\,e\,f^2-8\,c^3\,f^3\right)\right)\,\sqrt{c+d\,x^2}\right.$$
$$\left.\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]\right)\Big/\left(105\,d^3\,f^{3/2}\,\sqrt{\frac{e\,\left(c+d\,x^2\right)}{c\,\left(e+f\,x^2\right)}}\,\sqrt{e+f\,x^2}\right)+$$

$$\left(e^{3/2}\,\left(7\,a\,d\,f\,\left(9\,d\,e-c\,f\right)-b\,\left(3\,d^2\,e^2+9\,c\,d\,e\,f-4\,c^2\,f^2\right)\right)\,\sqrt{c+d\,x^2}\right.$$
$$\left.\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]\right)\Big/\left(105\,d^2\,f^{3/2}\,\sqrt{\frac{e\,\left(c+d\,x^2\right)}{c\,\left(e+f\,x^2\right)}}\,\sqrt{e+f\,x^2}\right)$$

Result (type 4, 372 leaves):

$$\frac{1}{105\,c^2\,\left(\frac{d}{c}\right)^{5/2}\,f^2\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}\left(-\sqrt{\frac{d}{c}}\,f\,x\,\left(c+d\,x^2\right)\,\left(e+f\,x^2\right)\right.$$
$$\left(4\,b\,c^2\,f^2-3\,b\,c\,d\,f\,\left(3\,e+f\,x^2\right)-7\,a\,d\,f\,\left(6\,d\,e+c\,f+3\,d\,f\,x^2\right)-3\,b\,d^2\,\left(e^2+8\,e\,f\,x^2+5\,f^2\,x^4\right)\right)-$$
$$i\,e\,\left(7\,a\,d\,f\,\left(3\,d^2\,e^2+7\,c\,d\,e\,f-2\,c^2\,f^2\right)+b\,\left(-6\,d^3\,e^3+9\,c\,d^2\,e^2\,f-19\,c^2\,d\,e\,f^2+8\,c^3\,f^3\right)\right)$$
$$\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]+$$
$$i\,e\,\left(-d\,e+c\,f\right)\,\left(-7\,a\,d\,f\,\left(3\,d\,e+c\,f\right)+b\,\left(6\,d^2\,e^2-6\,c\,d\,e\,f+4\,c^2\,f^2\right)\right)$$
$$\left.\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)$$

Problem 30: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x^2\right)\,\left(e+f\,x^2\right)^{3/2}}{\sqrt{c+d\,x^2}}\,dx$$

Optimal (type 4, 400 leaves, 6 steps):

$$\frac{\left(10\,a\,d\,f\,(2\,d\,e-c\,f)+b\,\left(3\,d^2\,e^2-13\,c\,d\,e\,f+8\,c^2\,f^2\right)\right)\,x\,\sqrt{c+d\,x^2}}{15\,d^3\,\sqrt{e+f\,x^2}}+\frac{(3\,b\,d\,e-4\,b\,c\,f+5\,a\,d\,f)\,x\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}{15\,d^2}+\frac{b\,x\,\sqrt{c+d\,x^2}\,\left(e+f\,x^2\right)^{3/2}}{5\,d}-\left(\sqrt{e}\,\left(10\,a\,d\,f\,(2\,d\,e-c\,f)+b\,\left(3\,d^2\,e^2-13\,c\,d\,e\,f+8\,c^2\,f^2\right)\right)\,\sqrt{c+d\,x^2}\;\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]\right)\Bigg/\left(15\,d^3\,\sqrt{f}\,\sqrt{\frac{e\,(c+d\,x^2)}{c\,(e+f\,x^2)}}\,\sqrt{e+f\,x^2}\right)+\left(e^{3/2}\,\left(5\,a\,d\,(3\,d\,e-c\,f)-b\,\left(6\,c\,d\,e-4\,c^2\,f\right)\right)\,\sqrt{c+d\,x^2}\;\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]\right)\Bigg/\left(15\,c\,d^2\,\sqrt{f}\,\sqrt{\frac{e\,(c+d\,x^2)}{c\,(e+f\,x^2)}}\,\sqrt{e+f\,x^2}\right)$$

Result (type 4, 275 leaves):

$$\left(-\sqrt{\frac{d}{c}}\,f\,x\,\left(c+d\,x^2\right)\,\left(e+f\,x^2\right)\,\left(4\,b\,c\,f-5\,a\,d\,f-3\,b\,d\,\left(2\,e+f\,x^2\right)\right)-i\,e\,\left(10\,a\,d\,f\,(2\,d\,e-c\,f)+b\,\left(3\,d^2\,e^2-13\,c\,d\,e\,f+8\,c^2\,f^2\right)\right)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]+i\,e\,(-d\,e+c\,f)\,(-3\,b\,d\,e+4\,b\,c\,f-5\,a\,d\,f)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)\Bigg/\left(15\,c^2\,\left(\frac{d}{c}\right)^{5/2}\,f\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}\right)$$

Problem 31: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x^2\right)\,\left(e+f\,x^2\right)^{3/2}}{\left(c+d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 369 leaves, 6 steps):

$$\frac{f\left(b c\left(7 d e-8 c f\right)-3 a d\left(d e-2 c f\right)\right) x \sqrt{c+d x^2}}{3 c d^3 \sqrt{e+f x^2}}+\frac{\left(4 b c-3 a d\right) f x \sqrt{c+d x^2} \sqrt{e+f x^2}}{3 c d^2}-\frac{\left(b c-a d\right) x \left(e+f x^2\right)^{3/2}}{c d \sqrt{c+d x^2}}-\left(\sqrt{e} \sqrt{f} \left(b c\left(7 d e-8 c f\right)-3 a d\left(d e-2 c f\right)\right) \sqrt{c+d x^2}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]\right) \Big/ \left(3 c d^3 \sqrt{\frac{e\left(c+d x^2\right)}{c\left(e+f x^2\right)}} \sqrt{e+f x^2}\right)+\left(e^{3/2}\left(3 b d e-4 b c f+3 a d f\right) \sqrt{c+d x^2}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]\right) \Big/ \left(3 c d^2 \sqrt{f} \sqrt{\frac{e\left(c+d x^2\right)}{c\left(e+f x^2\right)}} \sqrt{e+f x^2}\right)$$

Result (type 4, 248 leaves):

$$\left(\sqrt{\frac{d}{c}} \left(\sqrt{\frac{d}{c}}\, x \left(e+f x^2\right) \left(3 a d \left(d e-c f\right)+b c \left(-3 d e+4 c f+d f x^2\right)\right)+\right.\right.$$
$$i e \left(3 a d \left(d e-2 c f\right)+b c \left(-7 d e+8 c f\right)\right) \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}}$$
$$\text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right], \frac{c f}{d e}\right]-i \left(4 b c-3 a d\right) e \left(-d e+c f\right) \sqrt{1+\frac{d x^2}{c}}$$
$$\left.\left.\sqrt{1+\frac{f x^2}{e}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right], \frac{c f}{d e}\right]\right)\right) \Big/ \left(3 d^3 \sqrt{c+d x^2} \sqrt{e+f x^2}\right)$$

Problem 32: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^2\right) \left(e+f x^2\right)^{3/2}}{\left(c+d x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 373 leaves, 6 steps):

$$-\frac{f\left(b c\left(d e-8 c f\right)+2 a d\left(d e+c f\right)\right) x \sqrt{c+d x^2}}{3 c^2 d^3 \sqrt{e+f x^2}}+\frac{\left(b c\left(d e-4 c f\right)+a d\left(2 d e+c f\right)\right) x \sqrt{e+f x^2}}{3 c^2 d^2 \sqrt{c+d x^2}}-\frac{\left(b c-a d\right) x\left(e+f x^2\right)^{3/2}}{3 c d\left(c+d x^2\right)^{3/2}}+\left(\sqrt{e}\,\sqrt{f}\left(b c\left(d e-8 c f\right)+2 a d\left(d e+c f\right)\right)\sqrt{c+d x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]\right)\Bigg/\left(3 c^2 d^3\sqrt{\frac{e\left(c+d x^2\right)}{c\left(e+f x^2\right)}}\sqrt{e+f x^2}\right)+\frac{\left(4 b c-a d\right) e^{3/2}\sqrt{f}\,\sqrt{c+d x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{3 c^2 d^2\sqrt{\frac{e\left(c+d x^2\right)}{c\left(e+f x^2\right)}}\sqrt{e+f x^2}}$$

Result (type 4, 296 leaves):

$$\frac{1}{3 d^4\left(c+d x^2\right)^{3/2}\sqrt{e+f x^2}}\left(\frac{d}{c}\right)^{3/2}\left(\sqrt{\frac{d}{c}}\,x\left(e+f x^2\right)\left(b c\left(-4 c^2 f+d^2 e x^2-5 c d f x^2\right)+a d\left(c^2 f+2 d^2 e x^2+c d\left(3 e+2 f x^2\right)\right)\right)-\right.$$
$$i e\left(-2 a d\left(d e+c f\right)+b c\left(-d e+8 c f\right)\right)\left(c+d x^2\right)\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}$$
$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c f}{d e}\right]+i e\left(-a d\left(2 d e+c f\right)+b c\left(-d e+4 c f\right)\right)$$
$$\left.\left(c+d x^2\right)\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c f}{d e}\right]\right)$$

Problem 33: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b x^2\right)\left(e+f x^2\right)^{3/2}}{\left(c+d x^2\right)^{7/2}}\,dx$$

Optimal (type 4, 376 leaves, 5 steps):

$$\frac{(d\,(b\,c+4\,a\,d)\,e-c\,(4\,b\,c+a\,d)\,f)\,x\,\sqrt{e+f\,x^2}}{15\,c^2\,d^2\,\left(c+d\,x^2\right)^{3/2}}-\frac{(b\,c-a\,d)\,x\,\left(e+f\,x^2\right)^{3/2}}{5\,c\,d\,\left(c+d\,x^2\right)^{5/2}}+$$
$$\left(\left(b\,c\,\left(2\,d^2\,e^2+3\,c\,d\,e\,f-8\,c^2\,f^2\right)+a\,d\,\left(8\,d^2\,e^2-3\,c\,d\,e\,f-2\,c^2\,f^2\right)\right)\sqrt{e+f\,x^2}\right.$$
$$\left.\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{c\,f}{d\,e}\right]\right)\Big/\left(15\,c^{5/2}\,d^{5/2}\,(d\,e-c\,f)\,\sqrt{c+d\,x^2}\,\sqrt{\frac{c\,\left(e+f\,x^2\right)}{e\,\left(c+d\,x^2\right)}}\right)-$$
$$\left(e^{3/2}\,\sqrt{f}\,\left(b\,c\,(d\,e-4\,c\,f)+a\,d\,(4\,d\,e-c\,f)\right)\sqrt{c+d\,x^2}\;\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]\right)\Big/$$
$$\left(15\,c^3\,d^2\,(d\,e-c\,f)\,\sqrt{\frac{e\,\left(c+d\,x^2\right)}{c\,\left(e+f\,x^2\right)}}\,\sqrt{e+f\,x^2}\right)$$

Result (type 4, 382 leaves):

$$\frac{1}{15\,c^2\,d^3\,(d\,e-c\,f)\,\left(c+d\,x^2\right)^{5/2}\sqrt{e+f\,x^2}}\sqrt{\frac{d}{c}}\left(-\sqrt{\frac{d}{c}}\,x\,\left(e+f\,x^2\right)\right.$$
$$\left(3\,c^2\,(b\,c-a\,d)\,(d\,e-c\,f)^2-c\,(d\,e-c\,f)\,(b\,c\,(d\,e-7\,c\,f)+2\,a\,d\,(2\,d\,e+c\,f))\,\left(c+d\,x^2\right)+\right.$$
$$\left.\left(a\,d\,\left(-8\,d^2\,e^2+3\,c\,d\,e\,f+2\,c^2\,f^2\right)+b\,c\,\left(-2\,d^2\,e^2-3\,c\,d\,e\,f+8\,c^2\,f^2\right)\right)\left(c+d\,x^2\right)^2\right)-$$
$$i\,e\,\left(c+d\,x^2\right)^2\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\left(\left(a\,d\,\left(-8\,d^2\,e^2+3\,c\,d\,e\,f+2\,c^2\,f^2\right)+\right.\right.$$
$$\left.b\,c\,\left(-2\,d^2\,e^2-3\,c\,d\,e\,f+8\,c^2\,f^2\right)\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]+$$
$$\left.\left.(d\,e-c\,f)\,(a\,d\,(8\,d\,e+c\,f)+2\,b\,c\,(d\,e+2\,c\,f))\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)\right)$$

Problem 34: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x^2\right)\left(e+f\,x^2\right)^{3/2}}{\left(c+d\,x^2\right)^{9/2}}\,dx$$

Optimal (type 4, 531 leaves, 6 steps):

$$\frac{\left(d\,(b\,c+6\,a\,d)\,e-c\,(4\,b\,c+3\,a\,d)\,f\right)\,x\,\sqrt{e+f\,x^2}}{35\,c^2\,d^2\,\left(c+d\,x^2\right)^{5/2}}+$$

$$\left(\left(b\,c\,\left(4\,d^2\,e^2+c\,d\,e\,f-8\,c^2\,f^2\right)+3\,a\,d\,\left(8\,d^2\,e^2-5\,c\,d\,e\,f-2\,c^2\,f^2\right)\right)\,x\,\sqrt{e+f\,x^2}\right)\Big/$$

$$\left(105\,c^3\,d^2\,(d\,e-c\,f)\,\left(c+d\,x^2\right)^{3/2}\right)-\frac{(b\,c-a\,d)\,x\,\left(e+f\,x^2\right)^{3/2}}{7\,c\,d\,\left(c+d\,x^2\right)^{7/2}}+$$

$$\left(\left(6\,a\,d\,\left(8\,d^3\,e^3-12\,c\,d^2\,e^2\,f+2\,c^2\,d\,e\,f^2+c^3\,f^3\right)+b\,c\,\left(8\,d^3\,e^3-5\,c\,d^2\,e^2\,f-5\,c^2\,d\,e\,f^2+8\,c^3\,f^3\right)\right)\right.$$

$$\left.\sqrt{e+f\,x^2}\;\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\,1-\frac{c\,f}{d\,e}\right]\right)\Big/$$

$$\left(105\,c^{7/2}\,d^{5/2}\,(d\,e-c\,f)^2\,\sqrt{c+d\,x^2}\,\sqrt{\frac{c\,\left(e+f\,x^2\right)}{e\,\left(c+d\,x^2\right)}}\right)-$$

$$\left(e^{3/2}\,\sqrt{f}\,\left(3\,a\,d\,\left(8\,d^2\,e^2-11\,c\,d\,e\,f+c^2\,f^2\right)+2\,b\,c\,\left(2\,d^2\,e^2-c\,d\,e\,f+2\,c^2\,f^2\right)\right)\,\sqrt{c+d\,x^2}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],\,1-\frac{d\,e}{c\,f}\right]\right)\Big/\left(105\,c^4\,d^2\,(d\,e-c\,f)^2\,\sqrt{\frac{e\,\left(c+d\,x^2\right)}{c\,\left(e+f\,x^2\right)}}\,\sqrt{e+f\,x^2}\right)$$

Result (type 4, 545 leaves):

$$\frac{1}{105\,c^3\,d^3\,(d\,e-c\,f)^2\,\left(c+d\,x^2\right)^{7/2}\,\sqrt{e+f\,x^2}}\sqrt{\frac{d}{c}}\,\left(-\sqrt{\frac{d}{c}}\,x\,\left(e+f\,x^2\right)\right.$$

$$\left(15\,c^3\,(b\,c-a\,d)\,(d\,e-c\,f)^3-3\,c^2\,(d\,e-c\,f)^2\,\left(b\,c\,(d\,e-9\,c\,f)+2\,a\,d\,(3\,d\,e+c\,f)\right)\,\left(c+d\,x^2\right)-\right.$$
$$c\,(d\,e-c\,f)\,\left(b\,c\,\left(4\,d^2\,e^2+c\,d\,e\,f-8\,c^2\,f^2\right)+3\,a\,d\,\left(8\,d^2\,e^2-5\,c\,d\,e\,f-2\,c^2\,f^2\right)\right)\,\left(c+d\,x^2\right)^2-$$
$$\left(6\,a\,d\,\left(8\,d^3\,e^3-12\,c\,d^2\,e^2\,f+2\,c^2\,d\,e\,f^2+c^3\,f^3\right)+b\,c\,\left(8\,d^3\,e^3-5\,c\,d^2\,e^2\,f-5\,c^2\,d\,e\,f^2+8\,c^3\,f^3\right)\right)$$
$$\left.\left(c+d\,x^2\right)^3\right)+i\,e\,\left(c+d\,x^2\right)^3\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}$$

$$\left(\left(6\,a\,d\,\left(8\,d^3\,e^3-12\,c\,d^2\,e^2\,f+2\,c^2\,d\,e\,f^2+c^3\,f^3\right)+b\,c\,\left(8\,d^3\,e^3-5\,c\,d^2\,e^2\,f-5\,c^2\,d\,e\,f^2+8\,c^3\,f^3\right)\right)\right.$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]-(-d\,e+c\,f)\,\left(3\,a\,d\,\left(-16\,d^2\,e^2+16\,c\,d\,e\,f+c^2\,f^2\right)+\right.$$

$$\left.\left.\left.b\,c\,\left(-8\,d^2\,e^2+c\,d\,e\,f+4\,c^2\,f^2\right)\right)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]\right)\right)$$

Problem 35: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^2\right) \left(c+d x^2\right)^{5/2}}{\sqrt{e+f x^2}} \, dx$$

Optimal (type 4, 551 leaves, 7 steps):

$$\left(\left(7 a d f \left(8 d^2 e^2-23 c d e f+23 c^2 f^2\right)-b \left(48 d^3 e^3-128 c d^2 e^2 f+103 c^2 d e f^2-15 c^3 f^3\right)\right) x \sqrt{c+d x^2}\right) \Big/ \left(105 d f^3 \sqrt{e+f x^2}\right)-\frac{1}{105 f^3}\left(28 a d f (d e-2 c f)-b \left(24 d^2 e^2-43 c d e f+15 c^2 f^2\right)\right) x \sqrt{c+d x^2} \sqrt{e+f x^2}-\frac{(6 b d e-5 b c f-7 a d f) x \left(c+d x^2\right)^{3/2} \sqrt{e+f x^2}}{35 f^2}+\frac{b x \left(c+d x^2\right)^{5/2} \sqrt{e+f x^2}}{7 f}-\left(\sqrt{e} \left(7 a d f \left(8 d^2 e^2-23 c d e f+23 c^2 f^2\right)-b \left(48 d^3 e^3-128 c d^2 e^2 f+103 c^2 d e f^2-15 c^3 f^3\right)\right) \sqrt{c+d x^2}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]\right) \Big/ \left(105 d f^{7/2} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right)+\left(\sqrt{e} \left(7 a f \left(4 d^2 e^2-11 c d e f+15 c^2 f^2\right)-b e \left(24 d^2 e^2-61 c d e f+45 c^2 f^2\right)\right) \sqrt{c+d x^2}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]\right) \Big/ \left(105 f^{7/2} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right)$$

Result (type 4, 386 leaves):

$$\frac{1}{105 \sqrt{\frac{d}{c}} f^4 \sqrt{c+d x^2} \sqrt{e+f x^2}}\left(\sqrt{\frac{d}{c}} f x \left(c+d x^2\right) \left(e+f x^2\right) \left(7 a d f \left(-4 d e+11 c f+3 d f x^2\right)+b \left(45 c^2 f^2+c d f \left(-61 e+45 f x^2\right)+3 d^2 \left(8 e^2-6 e f x^2+5 f^2 x^4\right)\right)\right)-i e \left(7 a d f \left(8 d^2 e^2-23 c d e f+23 c^2 f^2\right)+b \left(-48 d^3 e^3+128 c d^2 e^2 f-103 c^2 d e f^2+15 c^3 f^3\right)\right) \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}}\, \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{d}{c}} x\right],\frac{c f}{d e}\right]+i (-d e+c f) \left(4 b e \left(12 d^2 e^2-26 c d e f+15 c^2 f^2\right)-7 a f \left(8 d^2 e^2-19 c d e f+15 c^2 f^2\right)\right) \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}}\, \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{d}{c}} x\right],\frac{c f}{d e}\right]\right)$$

Problem 36: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^2\right) \left(c+d x^2\right)^{3/2}}{\sqrt{e+f x^2}} \, dx$$

Optimal (type 4, 396 leaves, 6 steps):

$$-\frac{\left(10 a d f (d e-2 c f)-b \left(8 d^2 e^2-13 c d e f+3 c^2 f^2\right)\right) x \sqrt{c+d x^2}}{15 d f^2 \sqrt{e+f x^2}}-\frac{(4 b d e-3 b c f-5 a d f) x \sqrt{c+d x^2} \sqrt{e+f x^2}}{15 f^2}+\frac{b x \left(c+d x^2\right)^{3/2} \sqrt{e+f x^2}}{5 f}+\left(\sqrt{e} \left(10 a d f (d e-2 c f)-b \left(8 d^2 e^2-13 c d e f+3 c^2 f^2\right)\right) \sqrt{c+d x^2} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]\right) \Bigg/ \left(15 d f^{5/2} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right)-\left(\sqrt{e} (5 a f (d e-3 c f)-b (4 d e^2-6 c e f)) \sqrt{c+d x^2} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]\right) \Bigg/ \left(15 f^{5/2} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right)$$

Result (type 4, 279 leaves):

$$\frac{1}{15 \sqrt{\frac{d}{c}} f^3 \sqrt{c+d x^2} \sqrt{e+f x^2}}\left(\sqrt{\frac{d}{c}} f x \left(c+d x^2\right) \left(e+f x^2\right) \left(5 a d f+b \left(-4 d e+6 c f+3 d f x^2\right)\right)-i e \left(-10 a d f (d e-2 c f)+b \left(8 d^2 e^2-13 c d e f+3 c^2 f^2\right)\right) \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{d}{c}} x\right], \frac{c f}{d e}\right]+i (-d e+c f) (5 a f (2 d e-3 c f)+b e (-8 d e+9 c f)) \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{d}{c}} x\right], \frac{c f}{d e}\right]\right)$$

Problem 37: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^2\right) \sqrt{c+d x^2}}{\sqrt{e+f x^2}} \, dx$$

Optimal (type 4, 282 leaves, 5 steps):

$$-\frac{(2 b d e-b c f-3 a d f)\, x \sqrt{c+d x^2}}{3 d f \sqrt{e+f x^2}}+\frac{b x \sqrt{c+d x^2}\, \sqrt{e+f x^2}}{3 f}+\left(\sqrt{e}\, (2 b d e-b c f-3 a d f) \sqrt{c+d x^2}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]\right)\Big/\left(3 d f^{3/2} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}}\, \sqrt{e+f x^2}\right)-\frac{\sqrt{e}\, (b e-3 a f) \sqrt{c+d x^2}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{3 f^{3/2} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}}\, \sqrt{e+f x^2}}$$

Result (type 4, 215 leaves):

$$\left(b \sqrt{\frac{d}{c}}\, f x \left(c+d x^2\right) \left(e+f x^2\right)-i e (-2 b d e+b c f+3 a d f) \sqrt{1+\frac{d x^2}{c}}\, \sqrt{1+\frac{f x^2}{e}}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right],\frac{c f}{d e}\right]+i (2 b e-3 a f) (-d e+c f) \sqrt{1+\frac{d x^2}{c}}\, \sqrt{1+\frac{f x^2}{e}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right],\frac{c f}{d e}\right]\right)\Big/\left(3 \sqrt{\frac{d}{c}}\, f^2 \sqrt{c+d x^2}\, \sqrt{e+f x^2}\right)$$

Problem 38: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b x^2}{\sqrt{c+d x^2}\, \sqrt{e+f x^2}}\, dx$$

Optimal (type 4, 206 leaves, 4 steps):

$$\frac{b x \sqrt{c+d x^2}}{d \sqrt{e+f x^2}}-\frac{b \sqrt{e}\, \sqrt{c+d x^2}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{d \sqrt{f}\, \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}}\, \sqrt{e+f x^2}}+\frac{a \sqrt{e}\, \sqrt{c+d x^2}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{c \sqrt{f}\, \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}}\, \sqrt{e+f x^2}}$$

Result (type 4, 131 leaves):

$$-\left(\left(\left(i \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \left(b e \,\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{d}{c}} x\right], \frac{c f}{d e}\right] + \right.\right.\right.\right.$$

$$\left.\left.\left.(-b e + a f) \,\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{d}{c}} x\right], \frac{c f}{d e}\right]\right)\right) \Big/ \left(\sqrt{\frac{d}{c}} f \sqrt{c+d x^2} \sqrt{e+f x^2}\right)\right)$$

Problem 39: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b x^2}{\left(c+d x^2\right)^{3/2} \sqrt{e+f x^2}} \, dx$$

Optimal (type 4, 209 leaves, 3 steps):

$$-\frac{(b c - a d) \sqrt{e+f x^2} \,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1-\frac{c f}{d e}\right]}{\sqrt{c} \sqrt{d} (d e - c f) \sqrt{c+d x^2} \sqrt{\frac{c (e+f x^2)}{e (c+d x^2)}}} +$$

$$\frac{\sqrt{e} (b e - a f) \sqrt{c+d x^2} \,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]}{c \sqrt{f} (d e - c f) \sqrt{\frac{e (c+d x^2)}{c (e+f x^2)}} \sqrt{e+f x^2}}$$

Result (type 4, 206 leaves):

$$\left(\sqrt{\frac{d}{c}} \left(\sqrt{\frac{d}{c}} (b c - a d) x (e+f x^2) + \right.\right.$$

$$i (b c - a d) e \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \,\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{d}{c}} x\right], \frac{c f}{d e}\right] -$$

$$\left.\left. i a (-d e + c f) \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \,\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{d}{c}} x\right], \frac{c f}{d e}\right]\right)\right) \Big/$$

$$\left(d (-d e + c f) \sqrt{c+d x^2} \sqrt{e+f x^2}\right)$$

Problem 40: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b x^2}{\left(c+d x^2\right)^{5/2} \sqrt{e+f x^2}} \, dx$$

Optimal (type 4, 284 leaves, 4 steps):

$$-\frac{(b c-a d)\, x\, \sqrt{e+f x^2}}{3 c\, (d e-c f)\, (c+d x^2)^{3/2}}+$$

$$\left((2 a d\, (d e-2 c f)+b c\, (d e+c f))\, \sqrt{e+f x^2}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right],\, 1-\frac{c f}{d e}\right]\right)\Big/$$

$$\left(3 c^{3/2}\, \sqrt{d}\, (d e-c f)^2\, \sqrt{c+d x^2}\, \sqrt{\frac{c\, (e+f x^2)}{e\, (c+d x^2)}}\right)-$$

$$\left(\sqrt{e}\, \sqrt{f}\, (2 b c e+a d e-3 a c f)\, \sqrt{c+d x^2}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right],\, 1-\frac{d e}{c f}\right]\right)\Big/$$

$$\left(3 c^2\, (d e-c f)^2\, \sqrt{\frac{e\, (c+d x^2)}{c\, (e+f x^2)}}\, \sqrt{e+f x^2}\right)$$

Result (type 4, 302 leaves):

$$\frac{1}{3 c^2\, \sqrt{\frac{d}{c}}\, (d e-c f)^2\, (c+d x^2)^{3/2}\, \sqrt{e+f x^2}}$$

$$\left(\sqrt{\frac{d}{c}}\, x\, (e+f x^2)\, \left(b c\, (2 c^2 f+d^2 e x^2+c d f x^2)+a d\, (-5 c^2 f+2 d^2 e x^2+c d\, (3 e-4 f x^2))\right)+\right.$$

$$i\, e\, (2 a d\, (d e-2 c f)+b c\, (d e+c f))\, (c+d x^2)\, \sqrt{1+\frac{d x^2}{c}}\, \sqrt{1+\frac{f x^2}{e}}$$

$$\text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right],\, \frac{c f}{d e}\right]+i\, (-d e+c f)\, (b c e+2 a d e-3 a c f)$$

$$\left.(c+d x^2)\, \sqrt{1+\frac{d x^2}{c}}\, \sqrt{1+\frac{f x^2}{e}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right],\, \frac{c f}{d e}\right]\right)$$

Problem 41: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b x^2}{(c+d x^2)^{7/2}\, \sqrt{e+f x^2}}\, dx$$

Optimal (type 4, 401 leaves, 5 steps):

$$-\frac{(b c-a d)\, x \sqrt{e+f x^2}}{5 c\, (d e-c f)\, (c+d x^2)^{5/2}}+\frac{(4 a d\, (d e-2 c f)+b c\, (d e+3 c f))\, x \sqrt{e+f x^2}}{15 c^2\, (d e-c f)^2\, (c+d x^2)^{3/2}}+$$

$$\left((b c\, (2 d^2 e^2-7 c d e f-3 c^2 f^2)+a d\, (8 d^2 e^2-23 c d e f+23 c^2 f^2))\sqrt{e+f x^2}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right],1-\frac{c f}{d e}\right]\right)\Big/\left(15 c^{5/2}\sqrt{d}\, (d e-c f)^3\sqrt{c+d x^2}\sqrt{\frac{c\,(e+f x^2)}{e\,(c+d x^2)}}\right)-$$

$$\left(\sqrt{e}\,\sqrt{f}\,(b c e\,(d e-9 c f)+a\,(4 d^2 e^2-11 c d e f+15 c^2 f^2))\sqrt{c+d x^2}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]\right)\Big/\left(15 c^3\,(d e-c f)^3\sqrt{\frac{e\,(c+d x^2)}{c\,(e+f x^2)}}\sqrt{e+f x^2}\right)$$

Result (type 4, 393 leaves) :

$$\frac{1}{15 c^3\sqrt{\frac{d}{c}}\,(d e-c f)^3\,(c+d x^2)^{5/2}\sqrt{e+f x^2}}\left(-\sqrt{\frac{d}{c}}\, x\,(e+f x^2)\right.$$

$$\left(3 c^2\,(b c-a d)\,(d e-c f)^2+c\,(-d e+c f)\,(4 a d\,(d e-2 c f)+b c\,(d e+3 c f))\,(c+d x^2)+\right.$$

$$\left.(a d\,(-8 d^2 e^2+23 c d e f-23 c^2 f^2)+b c\,(-2 d^2 e^2+7 c d e f+3 c^2 f^2))\,(c+d x^2)^2\right)-$$

$$i\,(c+d x^2)^2\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\left(e\,(a d\,(-8 d^2 e^2+23 c d e f-23 c^2 f^2)+\right.$$

$$b c\,(-2 d^2 e^2+7 c d e f+3 c^2 f^2))\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right],\frac{c f}{d e}\right]+(d e-c f)$$

$$\left.\left.(2 b c e\,(d e-3 c f)+a\,(8 d^2 e^2-19 c d e f+15 c^2 f^2))\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right],\frac{c f}{d e}\right]\right)\right)$$

Problem 42: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(a+b x^2)\,(c+d x^2)^{5/2}}{(e+f x^2)^{3/2}}\,dx$$

Optimal (type 4, 501 leaves, 7 steps) :

$$-\frac{1}{15\,e\,f^3\sqrt{e+f x^2}}\left(5\,a\,f\left(8\,d^2 e^2-13\,c\,d\,e\,f+3\,c^2 f^2\right)-2\,b\,e\left(24\,d^2 e^2-44\,c\,d\,e\,f+19\,c^2 f^2\right)\right)x\sqrt{c+d x^2}-$$

$$\frac{(b\,e-a\,f)\,x\left(c+d x^2\right)^{5/2}}{e\,f\sqrt{e+f x^2}}-\frac{d\left(b\,e\,(24\,d\,e-23\,c\,f)-5\,a\,f\,(4\,d\,e-3\,c\,f)\right)x\sqrt{c+d x^2}\,\sqrt{e+f x^2}}{15\,e\,f^3}+$$

$$\frac{d\,(6\,b\,e-5\,a\,f)\,x\left(c+d x^2\right)^{3/2}\sqrt{e+f x^2}}{5\,e\,f^2}+$$

$$\left(\left(5\,a\,f\left(8\,d^2 e^2-13\,c\,d\,e\,f+3\,c^2 f^2\right)-2\,b\,e\left(24\,d^2 e^2-44\,c\,d\,e\,f+19\,c^2 f^2\right)\right)\sqrt{c+d x^2}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]\right)\Big/\left(15\sqrt{e}\,f^{7/2}\sqrt{\frac{e\left(c+d x^2\right)}{c\left(e+f x^2\right)}}\,\sqrt{e+f x^2}\right)-$$

$$\left(\sqrt{e}\left(10\,a\,d\,f\,(2\,d\,e-3\,c\,f)-b\left(24\,d^2 e^2-41\,c\,d\,e\,f+15\,c^2 f^2\right)\right)\sqrt{c+d x^2}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]\right)\Big/\left(15\,f^{7/2}\sqrt{\frac{e\left(c+d x^2\right)}{c\left(e+f x^2\right)}}\,\sqrt{e+f x^2}\right)$$

Result (type 4, 369 leaves):

$$\frac{1}{15\sqrt{\frac{d}{c}}\,e\,f^4\sqrt{c+d x^2}\,\sqrt{e+f x^2}}\left(\sqrt{\frac{d}{c}}\,f\,x\left(c+d x^2\right)\left(5\,a\,f\left(-6\,c\,d\,e\,f+3\,c^2 f^2+d^2 e\left(4\,e+f x^2\right)\right)+\right.\right.$$

$$\left.b\,e\left(-15\,c^2 f^2+c\,d\,f\left(41\,e+11\,f x^2\right)-3\,d^2\left(8\,e^2+2\,e\,f x^2-f^2 x^4\right)\right)\right)-$$

$$i\,d\,e\left(-5\,a\,f\left(8\,d^2 e^2-13\,c\,d\,e\,f+3\,c^2 f^2\right)+2\,b\,e\left(24\,d^2 e^2-44\,c\,d\,e\,f+19\,c^2 f^2\right)\right)$$

$$\sqrt{1+\frac{d x^2}{c}}\,\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]-$$

$$i\,e\,(-d\,e+c\,f)\left(5\,a\,d\,f\,(-8\,d\,e+9\,c\,f)+b\left(48\,d^2 e^2-64\,c\,d\,e\,f+15\,c^2 f^2\right)\right)$$

$$\left.\sqrt{1+\frac{d x^2}{c}}\,\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)$$

Problem 43: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b x^2\right)\left(c+d x^2\right)^{3/2}}{\left(e+f x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 358 leaves, 6 steps):

$$-\frac{\left(b e (8 d e-7 c f)-3 a f (2 d e-c f)\right) x \sqrt{c+d x^2}}{3 e f^2 \sqrt{e+f x^2}}-\frac{(b e-a f) x \left(c+d x^2\right)^{3/2}}{e f \sqrt{e+f x^2}}+\frac{d (4 b e-3 a f) x \sqrt{c+d x^2} \sqrt{e+f x^2}}{3 e f^2}+$$

$$\left(\left(b e (8 d e-7 c f)-3 a f (2 d e-c f)\right) \sqrt{c+d x^2}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]\right) \Big/ \left(3 \sqrt{e} f^{5/2} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right)-$$

$$\left(\sqrt{e} (4 b d e-3 b c f-3 a d f) \sqrt{c+d x^2}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]\right) \Big/ \left(3 f^{5/2} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right)$$

Result (type 4, 260 leaves):

$$\left(\sqrt{\frac{d}{c}} f x \left(c+d x^2\right) \left(3 a f (-d e+c f)+b e \left(4 d e-3 c f+d f x^2\right)\right)-\right.$$
$$i d e \left(-3 a f (-2 d e+c f)+b e (-8 d e+7 c f)\right) \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}}$$
$$\text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}} x\right], \frac{c f}{d e}\right]-i e (-d e+c f) (-8 b d e+3 b c f+6 a d f) \sqrt{1+\frac{d x^2}{c}}$$
$$\left.\sqrt{1+\frac{f x^2}{e}} \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}} x\right], \frac{c f}{d e}\right]\right) \Big/ \left(3 \sqrt{\frac{d}{c}} e f^3 \sqrt{c+d x^2} \sqrt{e+f x^2}\right)$$

Problem 44: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^2\right) \sqrt{c+d x^2}}{\left(e+f x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 258 leaves, 5 steps):

$$-\frac{(b e-a f)\, x\sqrt{c+d x^2}}{e f\sqrt{e+f x^2}}+\frac{(2 b e-a f)\, x\sqrt{c+d x^2}}{e f\sqrt{e+f x^2}}-\frac{(2 b e-a f)\sqrt{c+d x^2}\ \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{\sqrt{e}\, f^{3/2}\sqrt{\frac{e (c+d x^2)}{c (e+f x^2)}}\ \sqrt{e+f x^2}}+\frac{b\sqrt{e}\ \sqrt{c+d x^2}\ \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{f^{3/2}\sqrt{\frac{e (c+d x^2)}{c (e+f x^2)}}\ \sqrt{e+f x^2}}$$

Result (type 4, 208 leaves) :

$$\left(\sqrt{\frac{d}{c}}\ f(-b e+a f)\, x\,(c+d x^2)-\right.$$
$$i\, d e\,(2 b e-a f)\sqrt{1+\frac{d x^2}{c}}\ \sqrt{1+\frac{f x^2}{e}}\ \text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\ x\right],\frac{c f}{d e}\right]-$$
$$\left.i\, e\,(-2 b d e+b c f+a d f)\sqrt{1+\frac{d x^2}{c}}\ \sqrt{1+\frac{f x^2}{e}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\ x\right],\frac{c f}{d e}\right]\right)\Big/$$
$$\left(\sqrt{\frac{d}{c}}\ e f^2\sqrt{c+d x^2}\ \sqrt{e+f x^2}\right)$$

Problem 45: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b x^2}{\sqrt{c+d x^2}\ (e+f x^2)^{3/2}}\, dx$$

Optimal (type 4, 209 leaves, 3 steps) :

$$\frac{(b e-a f)\sqrt{c+d x^2}\ \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{\sqrt{e}\ \sqrt{f}\ (d e-c f)\sqrt{\frac{e (c+d x^2)}{c (e+f x^2)}}\ \sqrt{e+f x^2}}-\frac{(b c-a d)\sqrt{e}\ \sqrt{c+d x^2}\ \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{c\sqrt{f}\ (d e-c f)\sqrt{\frac{e (c+d x^2)}{c (e+f x^2)}}\ \sqrt{e+f x^2}}$$

Result (type 4, 212 leaves) :

$$\left(\sqrt{\frac{d}{c}}\, f\,(-b e+a f)\, x\,(c+d x^2)-\right.$$
$$i\, d\, e\,(b e-a f)\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c f}{d e}\right]-$$
$$\left.i\, b\, e\,(-d e+c f)\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c f}{d e}\right]\right)\Big/$$
$$\left(\sqrt{\frac{d}{c}}\, e\, f\,(-d e+c f)\sqrt{c+d x^2}\sqrt{e+f x^2}\right)$$

Problem 46: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b x^2}{(c+d x^2)^{3/2}\,(e+f x^2)^{3/2}}\,dx$$

Optimal (type 4, 272 leaves, 4 steps):

$$-\frac{(b c-a d)\, x}{c\,(d e-c f)\sqrt{c+d x^2}\sqrt{e+f x^2}}-$$
$$\left(\sqrt{f}\,(2 b c e-a d e-a c f)\sqrt{c+d x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]\right)\Big/$$
$$\left(c\sqrt{e}\,(d e-c f)^2\sqrt{\frac{e\,(c+d x^2)}{c\,(e+f x^2)}}\sqrt{e+f x^2}\right)+$$
$$\left(\sqrt{e}\,(b d e+b c f-2 a d f)\sqrt{c+d x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]\right)\Big/$$
$$\left(c\sqrt{f}\,(d e-c f)^2\sqrt{\frac{e\,(c+d x^2)}{c\,(e+f x^2)}}\sqrt{e+f x^2}\right)$$

Result (type 4, 262 leaves):

$$\left(\sqrt{\frac{d}{c}}\left(\sqrt{\frac{d}{c}}\,x\left(a\left(c^2 f^2+c\,d\,f^2 x^2+d^2 e\left(e+f x^2\right)\right)-b\,c\,e\left(c\,f+d\left(e+2 f x^2\right)\right)\right)-\right.\right.$$
$$i\,d\,e\left(2 b\,c\,e-a\left(d\,e+c\,f\right)\right)\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]-$$
$$\left.\left.i\left(b\,c-a\,d\right)e\left(-d\,e+c\,f\right)\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)\right)\Big/$$
$$\left(d\,e\left(d\,e-c\,f\right)^2\sqrt{c+d x^2}\sqrt{e+f x^2}\right)$$

Problem 47: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b x^2}{\left(c+d x^2\right)^{5/2}\left(e+f x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 375 leaves, 5 steps):

$$-\frac{\left(b\,c-a\,d\right)x}{3\,c\left(d\,e-c\,f\right)\left(c+d x^2\right)^{3/2}\sqrt{e+f x^2}}+\frac{\left(2 a\,d\left(d\,e-3 c\,f\right)+b\,c\left(d\,e+3 c\,f\right)\right)x}{3 c^2\left(d\,e-c\,f\right)^2\sqrt{c+d x^2}\sqrt{e+f x^2}}+$$
$$\left(\sqrt{f}\left(b\,c\,e\left(d\,e+7 c\,f\right)+a\left(2 d^2 e^2-7 c\,d\,e\,f-3 c^2 f^2\right)\right)\sqrt{c+d x^2}\right.$$
$$\left.\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]\right)\Big/\left(3 c^2\sqrt{e}\left(d\,e-c\,f\right)^3\sqrt{\frac{e\left(c+d x^2\right)}{c\left(e+f x^2\right)}}\sqrt{e+f x^2}\right)-$$
$$\left(\sqrt{e}\sqrt{f}\left(a\,d\left(d\,e-9 c\,f\right)+b\,c\left(5 d\,e+3 c\,f\right)\right)\sqrt{c+d x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]\right)\Big/$$
$$\left(3 c^2\left(d\,e-c\,f\right)^3\sqrt{\frac{e\left(c+d x^2\right)}{c\left(e+f x^2\right)}}\sqrt{e+f x^2}\right)$$

Result (type 4, 428 leaves):

$$\frac{1}{3 c^2 \sqrt{\frac{d}{c}}\, e\, (-d e+c f)^3 \left(c+d x^2\right)^{3/2} \sqrt{e+f x^2}}$$

$$\Bigg(\sqrt{\frac{d}{c}}\, x \Big(-b c e \left(3 c^3 f^2+d^3 e x^2 \left(e+f x^2\right)+c d^2 f x^2 \left(4 e+7 f x^2\right)+c^2 d f \left(5 e+11 f x^2\right)\right)+$$
$$a \left(3 c^4 f^3+6 c^3 d f^3 x^2-2 d^4 e^2 x^2 \left(e+f x^2\right)+c^2 d^2 f \left(8 e^2+8 e f x^2+3 f^2 x^4\right)+c d^3 e \left(-3 e^2+4 e f x^2+7 f^2 x^4\right)\right)\Big)-$$
$$i d e \left(b c e (d e+7 c f)+a \left(2 d^2 e^2-7 c d e f-3 c^2 f^2\right)\right) \left(c+d x^2\right) \sqrt{1+\frac{d x^2}{c}}$$
$$\sqrt{1+\frac{f x^2}{e}}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right], \frac{c f}{d e}\right]-$$
$$i e (-d e+c f) (2 a d (d e-3 c f)+b c (d e+3 c f)) \left(c+d x^2\right)$$
$$\sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right], \frac{c f}{d e}\right]\Bigg)$$

Problem 48: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+f x^2}{\sqrt{a+b x^2} \left(c+d x^2\right)^{3/2}}\, dx$$

Optimal (type 4, 209 leaves, 3 steps):

$$-\frac{(d e-c f) \sqrt{a+b x^2}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], 1-\frac{b c}{a d}\right]}{\sqrt{c} \sqrt{d} (b c-a d) \sqrt{\frac{c \left(a+b x^2\right)}{a \left(c+d x^2\right)}} \sqrt{c+d x^2}}+$$
$$\frac{\sqrt{c} (b e-a f) \sqrt{a+b x^2}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], 1-\frac{b c}{a d}\right]}{a \sqrt{d} (b c-a d) \sqrt{\frac{c \left(a+b x^2\right)}{a \left(c+d x^2\right)}} \sqrt{c+d x^2}}$$

Result (type 4, 212 leaves):

$$\left(\sqrt{\frac{b}{a}}\, d\, (d e - c f)\, x\, (a + b x^2) - i\, b\, c\, (-d e + c f) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right], \frac{a d}{b c}\right] - i\, c\, (-b c + a d)\, f \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right], \frac{a d}{b c}\right]\right) \Big/ \left(\sqrt{\frac{b}{a}}\, c\, d\, (-b c + a d) \sqrt{a + b x^2} \sqrt{c + d x^2}\right)$$

Problem 49: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x^2}{\sqrt{a - b x^2}\, (c + d x^2)^{3/2}}\, dx$$

Optimal (type 4, 247 leaves, 8 steps):

$$\frac{(d e - c f)\, x \sqrt{a - b x^2}}{c\, (b c + a d) \sqrt{c + d x^2}} + \left(\sqrt{a}\, \sqrt{b}\, (d e - c f) \sqrt{1 - \frac{b x^2}{a}} \sqrt{c + d x^2}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b}\, x}{\sqrt{a}}\right], -\frac{a d}{b c}\right]\right) \Big/ \left(c\, d\, (b c + a d) \sqrt{a - b x^2} \sqrt{1 + \frac{d x^2}{c}}\right) + \frac{\sqrt{a}\, f \sqrt{1 - \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b}\, x}{\sqrt{a}}\right], -\frac{a d}{b c}\right]}{\sqrt{b}\, d \sqrt{a - b x^2} \sqrt{c + d x^2}}$$

Result (type 4, 220 leaves):

$$\left(\sqrt{-\frac{b}{a}}\, d\,(d e-c f)\, x\,(a-b x^2)+\right.$$
$$i\, b\, c\,(-d e+c f)\sqrt{1-\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b}{a}}\, x\right],-\frac{a d}{b c}\right]-$$
$$\left. i\, c\,(b c+a d)\, f\sqrt{1-\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b}{a}}\, x\right],-\frac{a d}{b c}\right]\right)\Big/$$
$$\left(\sqrt{-\frac{b}{a}}\, c\, d\,(b c+a d)\sqrt{a-b x^2}\sqrt{c+d x^2}\right)$$

Problem 50: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+f x^2}{\sqrt{a+b x^2}\,(c-d x^2)^{3/2}}\, dx$$

Optimal (type 4, 237 leaves, 8 steps):

$$\frac{(d e+c f)\, x\sqrt{a+b x^2}}{c\,(b c+a d)\sqrt{c-d x^2}}-\frac{(d e+c f)\sqrt{a+b x^2}\sqrt{1-\frac{d x^2}{c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right],-\frac{b c}{a d}\right]}{\sqrt{c}\sqrt{d}\,(b c+a d)\sqrt{1+\frac{b x^2}{a}}\sqrt{c-d x^2}}+$$
$$\frac{e\sqrt{1+\frac{b x^2}{a}}\sqrt{1-\frac{d x^2}{c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right],-\frac{b c}{a d}\right]}{\sqrt{c}\sqrt{d}\sqrt{a+b x^2}\sqrt{c-d x^2}}$$

Result (type 4, 213 leaves):

$$\left(\sqrt{\frac{b}{a}}\, d\,(d e+c f)\, x\,(a+b x^2)-\right.$$
$$i\, b\, c\,(d e+c f)\sqrt{1+\frac{b x^2}{a}}\sqrt{1-\frac{d x^2}{c}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right],-\frac{a d}{b c}\right]+$$
$$\left. i\, c\,(b c+a d)\, f\sqrt{1+\frac{b x^2}{a}}\sqrt{1-\frac{d x^2}{c}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{b}{a}}\, x\right],-\frac{a d}{b c}\right]\right)\Big/$$
$$\left(\sqrt{\frac{b}{a}}\, c\, d\,(b c+a d)\sqrt{a+b x^2}\sqrt{c-d x^2}\right)$$

Problem 51: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+f x^2}{\sqrt{a-b x^2}\,\left(c-d x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 242 leaves, 8 steps):

$$-\frac{(d e+c f)\,x\,\sqrt{a-b x^2}}{c\,(b c-a d)\,\sqrt{c-d x^2}}+\frac{(d e+c f)\,\sqrt{a-b x^2}\,\sqrt{1-\frac{d x^2}{c}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b c}{a d}\right]}{\sqrt{c}\,\sqrt{d}\,(b c-a d)\,\sqrt{1-\frac{b x^2}{a}}\,\sqrt{c-d x^2}}+\frac{e\,\sqrt{1-\frac{b x^2}{a}}\,\sqrt{1-\frac{d x^2}{c}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b c}{a d}\right]}{\sqrt{c}\,\sqrt{d}\,\sqrt{a-b x^2}\,\sqrt{c-d x^2}}$$

Result (type 4, 221 leaves):

$$\left(\sqrt{-\frac{b}{a}}\,d\,(d e+c f)\,x\,\left(a-b x^2\right)+i\,b\,c\,(d e+c f)\,\sqrt{1-\frac{b x^2}{a}}\,\sqrt{1-\frac{d x^2}{c}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b}{a}}\,x\right],\frac{a d}{b c}\right]+i\,c\,(-b c+a d)\,f\,\sqrt{1-\frac{b x^2}{a}}\,\sqrt{1-\frac{d x^2}{c}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{b}{a}}\,x\right],\frac{a d}{b c}\right]\right)\Big/\left(\sqrt{-\frac{b}{a}}\,c\,d\,(-b c+a d)\,\sqrt{a-b x^2}\,\sqrt{c-d x^2}\right)$$

Problem 52: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b x^2}{\sqrt{2+d x^2}\,\sqrt{3+f x^2}}\,dx$$

Optimal (type 4, 191 leaves, 4 steps):

$$\frac{b\,x\,\sqrt{2+d\,x^2}}{d\,\sqrt{3+f\,x^2}}-\frac{\sqrt{2}\,b\,\sqrt{2+d\,x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{3}}\right],1-\frac{3\,d}{2\,f}\right]}{d\,\sqrt{f}\,\sqrt{\frac{2+d\,x^2}{3+f\,x^2}}\,\sqrt{3+f\,x^2}}+$$

$$\frac{a\,\sqrt{2+d\,x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{3}}\right],1-\frac{3\,d}{2\,f}\right]}{\sqrt{2}\,\sqrt{f}\,\sqrt{\frac{2+d\,x^2}{3+f\,x^2}}\,\sqrt{3+f\,x^2}}$$

Result (type 4, 81 leaves):

$$-\frac{1}{\sqrt{3}\,\sqrt{d}\,f}\,\mathrm{i}\left(3\,b\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{d}\,x}{\sqrt{2}}\right],\frac{2\,f}{3\,d}\right]+\left(-3\,b+a\,f\right)\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{d}\,x}{\sqrt{2}}\right],\frac{2\,f}{3\,d}\right]\right)$$

Problem 53: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x^2\right)\,\sqrt{2+d\,x^2}}{\sqrt{3+f\,x^2}}\,dx$$

Optimal (type 4, 262 leaves, 5 steps):

$$-\frac{\left(6\,b\,d-2\,b\,f-3\,a\,d\,f\right)\,x\,\sqrt{2+d\,x^2}}{3\,d\,f\,\sqrt{3+f\,x^2}}+\frac{b\,x\,\sqrt{2+d\,x^2}\,\sqrt{3+f\,x^2}}{3\,f}+$$

$$\left(\sqrt{2}\,\left(6\,b\,d-2\,b\,f-3\,a\,d\,f\right)\,\sqrt{2+d\,x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{3}}\right],1-\frac{3\,d}{2\,f}\right]\right)\Big/$$

$$\left(3\,d\,f^{3/2}\,\sqrt{\frac{2+d\,x^2}{3+f\,x^2}}\,\sqrt{3+f\,x^2}\right)-\frac{\sqrt{2}\,\left(b-a\,f\right)\,\sqrt{2+d\,x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{3}}\right],1-\frac{3\,d}{2\,f}\right]}{f^{3/2}\,\sqrt{\frac{2+d\,x^2}{3+f\,x^2}}\,\sqrt{3+f\,x^2}}$$

Result (type 4, 142 leaves):

$$\frac{1}{3\,\sqrt{d}\,f^2}\left(b\,\sqrt{d}\,f\,x\,\sqrt{2+d\,x^2}\,\sqrt{3+f\,x^2}+\mathrm{i}\,\sqrt{3}\,\left(6\,b\,d-2\,b\,f-3\,a\,d\,f\right)\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{d}\,x}{\sqrt{2}}\right],\frac{2\,f}{3\,d}\right]+\right.$$

$$\left.\mathrm{i}\,\sqrt{3}\,\left(3\,d-2\,f\right)\,\left(-2\,b+a\,f\right)\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{\sqrt{d}\,x}{\sqrt{2}}\right],\frac{2\,f}{3\,d}\right]\right)$$

Problem 54: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(a+b\,x^2\right)\,\sqrt{2+d\,x^2}\,\sqrt{3+f\,x^2}\,dx$$

Optimal (type 4, 356 leaves, 6 steps):

$$\frac{\left(5\,a\,d\,f\,(3\,d+2\,f)-2\,b\,(9\,d^2-6\,d\,f+4\,f^2)\right)\,x\,\sqrt{2+d\,x^2}}{15\,d^2\,f\,\sqrt{3+f\,x^2}}+\frac{(3\,b\,d-4\,b\,f+5\,a\,d\,f)\,x\,\sqrt{2+d\,x^2}\,\sqrt{3+f\,x^2}}{15\,d\,f}+\frac{b\,x\,(2+d\,x^2)^{3/2}\,\sqrt{3+f\,x^2}}{5\,d}-$$
$$\left(\sqrt{2}\,\left(5\,a\,d\,f\,(3\,d+2\,f)-2\,b\,(9\,d^2-6\,d\,f+4\,f^2)\right)\,\sqrt{2+d\,x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{3}}\right],1-\frac{3\,d}{2\,f}\right]\right)\Big/$$
$$\left(15\,d^2\,f^{3/2}\,\sqrt{\frac{2+d\,x^2}{3+f\,x^2}}\,\sqrt{3+f\,x^2}\right)-$$
$$\left(\sqrt{2}\,(3\,b\,d+2\,b\,f-10\,a\,d\,f)\,\sqrt{2+d\,x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{3}}\right],1-\frac{3\,d}{2\,f}\right]\right)\Big/$$
$$\left(5\,d\,f^{3/2}\,\sqrt{\frac{2+d\,x^2}{3+f\,x^2}}\,\sqrt{3+f\,x^2}\right)$$

Result (type 4, 186 leaves):

$$\frac{1}{15\,d^{3/2}\,f^2}\left(\sqrt{d}\,f\,x\,\sqrt{2+d\,x^2}\,\sqrt{3+f\,x^2}\,\left(2\,b\,f+5\,a\,d\,f+3\,b\,d\,(1+f\,x^2)\right)+\right.$$
$$i\,\sqrt{3}\,\left(-5\,a\,d\,f\,(3\,d+2\,f)+2\,b\,(9\,d^2-6\,d\,f+4\,f^2)\right)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{d}\,x}{\sqrt{2}}\right],\frac{2\,f}{3\,d}\right]+$$
$$\left.i\,\sqrt{3}\,(3\,d-2\,f)\,(-6\,b\,d+2\,b\,f+5\,a\,d\,f)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{d}\,x}{\sqrt{2}}\right],\frac{2\,f}{3\,d}\right]\right)$$

Problem 55: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{-b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{\sqrt{1+\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}}\,dx$$

Optimal (type 4, 113 leaves, 2 steps):

$$-\frac{1}{\sqrt{2}\,\sqrt{c}}$$
$$\sqrt{b-\sqrt{b^2-4\,a\,c}}\,\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4\,a\,c}}}\right],\frac{b-\sqrt{b^2-4\,a\,c}}{b+\sqrt{b^2-4\,a\,c}}\right]$$

Result (type 4, 104 leaves):

$$-2\,i\,\sqrt{2}\,a\,\sqrt{\frac{c}{-b+\sqrt{b^2-4\,a\,c}}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{-b+\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b-\sqrt{b^2-4\,a\,c}}{b+\sqrt{b^2-4\,a\,c}}\right]$$

Problem 56: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}\,dx$$

Optimal (type 4, 526 leaves, 5 steps):

$$\frac{\left(b-\sqrt{b^2-4\,a\,c}\right)x\,\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}}{\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}-\left(\left(b-\sqrt{b^2-4\,a\,c}\right)\sqrt{b+\sqrt{b^2-4\,a\,c}}\right.$$

$$\left.\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\;\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4\,a\,c}}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{c}\,\sqrt{\frac{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\right)+\left(\left(b-\sqrt{b^2-4\,a\,c}\right)\sqrt{b+\sqrt{b^2-4\,a\,c}}\right.$$

$$\left.\sqrt{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\;\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4\,a\,c}}}\right],\,-\frac{2\,\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$

$$\left(\sqrt{2}\,\sqrt{c}\,\sqrt{\frac{1+\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}\,\sqrt{1+\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\right)$$

Result (type 4, 203 leaves):

$$-\frac{1}{\sqrt{2}\,\sqrt{\frac{c}{b-\sqrt{b^2-4\,a\,c}}}}$$

$$i\left(\left(b+\sqrt{b^2-4\,a\,c}\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b-\sqrt{b^2-4\,a\,c}}{b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.2\,\sqrt{b^2-4\,a\,c}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b-\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b-\sqrt{b^2-4\,a\,c}}{b+\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 64: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(c+d x^2\right)^{5/2} \sqrt{e+f x^2}}{a+b x^2} \, dx$$

Optimal (type 4, 608 leaves, 14 steps):

$$\frac{d \left(7 c e-\frac{2 d e^2}{f}+\frac{3 c^2 f}{d}\right) x \sqrt{c+d x^2}}{15 b \sqrt{e+f x^2}}+\frac{(b c-a d) (b d e+4 b c f-3 a d f) x \sqrt{c+d x^2}}{3 b^3 \sqrt{e+f x^2}}+\frac{d (b c-a d) x \sqrt{c+d x^2} \sqrt{e+f x^2}}{3 b^2}-\frac{2 d (d e-3 c f) x \sqrt{c+d x^2} \sqrt{e+f x^2}}{15 b f}+\frac{d^2 x \sqrt{c+d x^2} \left(e+f x^2\right)^{3/2}}{5 b f}-$$

$$\left(\sqrt{e} \left(15 a^2 d^2 f^2-5 a b d f (d e+7 c f)+b^2 \left(-2 d^2 e^2+12 c d e f+23 c^2 f^2\right)\right) \sqrt{c+d x^2} \, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]\right) \Bigg/ \left(15 b^3 f^{3/2} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right)+$$

$$\left(d e^{3/2} \left(-40 a b c d f+15 a^2 d^2 f+b^2 c (-d e+34 c f)\right) \sqrt{c+d x^2} \, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]\right) \Bigg/ \left(15 b^3 c f^{3/2} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right)+$$

$$\frac{(b c-a d)^3 e^{3/2} \sqrt{c+d x^2} \, \text{EllipticPi}\left[1-\frac{b e}{a f}, \text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]}{a b^3 c \sqrt{f} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}}$$

Result (type 4, 456 leaves):

$$\frac{1}{15\,a\,b^4\,\sqrt{\frac{d}{c}}\,f^2\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}$$

$$\left(-i\,a\,b\,d\,e\,\left(15\,a^2\,d^2\,f^2-5\,a\,b\,d\,f\,\left(d\,e+7\,c\,f\right)+b^2\,\left(-2\,d^2\,e^2+12\,c\,d\,e\,f+23\,c^2\,f^2\right)\right)\,\sqrt{1+\frac{d\,x^2}{c}}\right.$$

$$\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]-i\,a\,\left(45\,a^2\,b\,c\,d^2\,f^3-15\,a^3\,d^3\,f^3+\right.$$

$$\left.5\,a\,b^2\,d\,f\,\left(d^2\,e^2-c\,d\,e\,f-9\,c^2\,f^2\right)+b^3\,\left(2\,d^3\,e^3-13\,c\,d^2\,e^2\,f+11\,c^2\,d\,e\,f^2+15\,c^3\,f^3\right)\right)$$

$$\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]+$$

$$f\,\left(a\,b^2\,d\,\sqrt{\frac{d}{c}}\,x\,\left(c+d\,x^2\right)\,\left(e+f\,x^2\right)\,\left(11\,b\,c\,f-5\,a\,d\,f+b\,d\,\left(e+3\,f\,x^2\right)\right)-15\,i\,\left(b\,c-a\,d\right)^3\right.$$

$$\left.\left.f\,\left(b\,e-a\,f\right)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},\,i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]\right)\right)$$

Problem 65: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(c+d\,x^2\right)^{3/2}\,\sqrt{e+f\,x^2}}{a+b\,x^2}\,dx$$

Optimal (type 4, 400 leaves, 7 steps):

$$\frac{\left(b\,d\,e+4\,b\,c\,f-3\,a\,d\,f\right)\,x\,\sqrt{c+d\,x^2}}{3\,b^2\,\sqrt{e+f\,x^2}}+\frac{d\,x\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}{3\,b}-\left(\sqrt{e}\,\left(b\,d\,e+4\,b\,c\,f-3\,a\,d\,f\right)\,\sqrt{c+d\,x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]\right)\Big/\left(3\,b^2\,\sqrt{f}\,\sqrt{\frac{e\,\left(c+d\,x^2\right)}{c\,\left(e+f\,x^2\right)}}\,\sqrt{e+f\,x^2}\right)+\frac{d\,\left(5\,b\,c-3\,a\,d\right)\,e^{3/2}\,\sqrt{c+d\,x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]}{3\,b^2\,c\,\sqrt{f}\,\sqrt{\frac{e\,\left(c+d\,x^2\right)}{c\,\left(e+f\,x^2\right)}}\,\sqrt{e+f\,x^2}}+\frac{\left(b\,c-a\,d\right)^2\,e^{3/2}\,\sqrt{c+d\,x^2}\,\text{EllipticPi}\left[1-\frac{b\,e}{a\,f},\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]}{a\,b^2\,c\,\sqrt{f}\,\sqrt{\frac{e\,\left(c+d\,x^2\right)}{c\,\left(e+f\,x^2\right)}}\,\sqrt{e+f\,x^2}}$$

Result (type 4, 346 leaves):

$$\frac{1}{3\,a\,b^3\,\sqrt{\frac{d}{c}}\,f\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}\left(-i\,a\,b\,d\,e\,\left(b\,d\,e+4\,b\,c\,f-3\,a\,d\,f\right)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]-i\,a\,\left(-6\,a\,b\,c\,d\,f^2+3\,a^2\,d^2\,f^2+b^2\,\left(-d^2\,e^2+c\,d\,e\,f+3\,c^2\,f^2\right)\right)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]+f\left(a\,b^2\,d\,\sqrt{\frac{d}{c}}\,x\,\left(c+d\,x^2\right)\,\left(e+f\,x^2\right)-3\,i\,\left(b\,c-a\,d\right)^2\,\left(b\,e-a\,f\right)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)\right)$$

Problem 66: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}{a+b\,x^2}\,dx$$

Optimal (type 4, 321 leaves, 6 steps):

$$\frac{f x \sqrt{c+d x^2}}{b \sqrt{e+f x^2}} - \frac{\sqrt{e}\,\sqrt{f}\,\sqrt{c+d x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{b \sqrt{\frac{e\,(c+d x^2)}{c\,(e+f x^2)}}\,\sqrt{e+f x^2}} +$$

$$\frac{d\,e^{3/2}\sqrt{c+d x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{b\,c\,\sqrt{f}\,\sqrt{\frac{e\,(c+d x^2)}{c\,(e+f x^2)}}\,\sqrt{e+f x^2}} +$$

$$\frac{(b c-a d)\,e^{3/2}\sqrt{c+d x^2}\,\text{EllipticPi}\left[1-\frac{b e}{a f},\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{a\,b\,c\,\sqrt{f}\,\sqrt{\frac{e\,(c+d x^2)}{c\,(e+f x^2)}}\,\sqrt{e+f x^2}}$$

Result (type 4, 184 leaves):

$$-\left(\left(\mathrm{i}\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\left(a b d e\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c f}{d e}\right]+\right.\right.\right.$$

$$(b c-a d)\left(a f\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c f}{d e}\right]+(b e-a f)\right.$$

$$\left.\left.\left.\left.\text{EllipticPi}\left[\frac{b c}{a d},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c f}{d e}\right]\right)\right)\right)\Big/\left(a b^2\sqrt{\frac{d}{c}}\,\sqrt{c+d x^2}\,\sqrt{e+f x^2}\right)\right)$$

Problem 67: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{e+f x^2}}{(a+b x^2)\,\sqrt{c+d x^2}}\,dx$$

Optimal (type 4, 102 leaves, 1 step):

$$\frac{e^{3/2}\sqrt{c+d x^2}\,\text{EllipticPi}\left[1-\frac{b e}{a f},\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{a\,c\,\sqrt{f}\,\sqrt{\frac{e\,(c+d x^2)}{c\,(e+f x^2)}}\,\sqrt{e+f x^2}}$$

Result (type 4, 143 leaves):

$$-\left(\left(\mathrm{i}\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\left(a f\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c f}{d e}\right]+\right.\right.\right.$$

$$\left.\left.\left.(b e-a f)\,\text{EllipticPi}\left[\frac{b c}{a d},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c f}{d e}\right]\right)\right)\Big/\left(a b\sqrt{\frac{d}{c}}\,\sqrt{c+d x^2}\,\sqrt{e+f x^2}\right)\right)$$

Problem 68: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{e+f x^2}}{\left(a+b x^2\right) \left(c+d x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 209 leaves, 3 steps):

$$-\frac{\sqrt{d} \sqrt{e+f x^2} \text{ EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1-\frac{c f}{d e}\right]}{\sqrt{c} (b c-a d) \sqrt{c+d x^2} \sqrt{\frac{c (e+f x^2)}{e (c+d x^2)}}}+\frac{b e^{3/2} \sqrt{c+d x^2} \text{ EllipticPi}\left[1-\frac{b e}{a f}, \text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]}{a c (b c-a d) \sqrt{f} \sqrt{\frac{e (c+d x^2)}{c (e+f x^2)}} \sqrt{e+f x^2}}$$

Result (type 4, 347 leaves):

$$\frac{1}{a d (-b c+a d) \sqrt{c+d x^2} \sqrt{e+f x^2} \sqrt{\frac{d}{c}}} \left(a d \sqrt{\frac{d}{c}} e x+a d \sqrt{\frac{d}{c}} f x^3+i a d e \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \text{ EllipticE}\left[i \text{ ArcSinh}\left[\sqrt{\frac{d}{c}} x\right], \frac{c f}{d e}\right]+\right.$$
$$i a (-d e+c f) \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \text{ EllipticF}\left[i \text{ ArcSinh}\left[\sqrt{\frac{d}{c}} x\right], \frac{c f}{d e}\right]+$$
$$i b c e \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \text{ EllipticPi}\left[\frac{b c}{a d}, i \text{ ArcSinh}\left[\sqrt{\frac{d}{c}} x\right], \frac{c f}{d e}\right]-$$
$$\left. i a c f \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \text{ EllipticPi}\left[\frac{b c}{a d}, i \text{ ArcSinh}\left[\sqrt{\frac{d}{c}} x\right], \frac{c f}{d e}\right]\right)$$

Problem 69: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{e+f x^2}}{\left(a+b x^2\right) \left(c+d x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 401 leaves, 6 steps):

$$-\frac{d\,x\,\sqrt{e+f\,x^2}}{3\,c\,(b\,c-a\,d)\,(c+d\,x^2)^{3/2}}-\left(\sqrt{d}\,\left(b\,c\,(5\,d\,e-4\,c\,f)-a\,d\,(2\,d\,e-c\,f)\right)\sqrt{e+f\,x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{c\,f}{d\,e}\right]\right)\Big/\left(3\,c^{3/2}\,(b\,c-a\,d)^2\,(d\,e-c\,f)\,\sqrt{c+d\,x^2}\,\sqrt{\frac{c\,(e+f\,x^2)}{e\,(c+d\,x^2)}}\right)+$$

$$\frac{d\,e^{3/2}\,\sqrt{f}\,\sqrt{c+d\,x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]}{3\,c^2\,(b\,c-a\,d)\,(d\,e-c\,f)\,\sqrt{\frac{e\,(c+d\,x^2)}{c\,(e+f\,x^2)}}\,\sqrt{e+f\,x^2}}+$$

$$\frac{b^2\,e^{3/2}\,\sqrt{c+d\,x^2}\,\text{EllipticPi}\left[1-\frac{b\,e}{a\,f},\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]}{a\,c\,(b\,c-a\,d)^2\,\sqrt{f}\,\sqrt{\frac{e\,(c+d\,x^2)}{c\,(e+f\,x^2)}}\,\sqrt{e+f\,x^2}}$$

Result (type 4, 427 leaves) :

$$\frac{1}{3\,a\,c^2\,\sqrt{\frac{d}{c}}\,(b\,c-a\,d)^2\,(-d\,e+c\,f)\,(c+d\,x^2)^{3/2}\,\sqrt{e+f\,x^2}}\left(a\,c\,\left(\frac{d}{c}\right)^{3/2}x\,(e+f\,x^2)\right.$$
$$\left(b\,c\,(6\,c\,d\,e-5\,c^2\,f+5\,d^2\,e\,x^2-4\,c\,d\,f\,x^2)+a\,d\,(-3\,c\,d\,e+2\,c^2\,f-2\,d^2\,e\,x^2+c\,d\,f\,x^2)\right)-$$
$$i\,a\,d\,e\,\left(a\,d\,(2\,d\,e-c\,f)+b\,c\,(-5\,d\,e+4\,c\,f)\right)(c+d\,x^2)\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}$$
$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]-i\,a\,(-d\,e+c\,f)\,\left(2\,a\,d^2\,e+b\,c\,(-5\,d\,e+3\,c\,f)\right)$$
$$(c+d\,x^2)\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]-3\,i\,b\,c^2\,(b\,e-a\,f)$$
$$\left.(-d\,e+c\,f)\,(c+d\,x^2)\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)$$

Problem 70: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{e+f\,x^2}}{(a+b\,x^2)\,(c+d\,x^2)^{7/2}}\,dx$$

Optimal (type 4, 630 leaves, 9 steps) :

$$-\frac{d\,x\,\sqrt{e+f\,x^2}}{5\,c\,(b\,c-a\,d)\,(c+d\,x^2)^{5/2}}-\frac{d\,(b\,c\,(9\,d\,e-8\,c\,f)-a\,d\,(4\,d\,e-3\,c\,f))\,x\,\sqrt{e+f\,x^2}}{15\,c^2\,(b\,c-a\,d)^2\,(d\,e-c\,f)\,(c+d\,x^2)^{3/2}}-$$

$$\frac{b^2\,\sqrt{d}\,\sqrt{e+f\,x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{c\,f}{d\,e}\right]}{\sqrt{c}\,(b\,c-a\,d)^3\,\sqrt{c+d\,x^2}\,\sqrt{\frac{c\,(e+f\,x^2)}{e\,(c+d\,x^2)}}}+$$

$$\left(\sqrt{d}\,\left(a\,d\,(8\,d^2\,e^2-13\,c\,d\,e\,f+3\,c^2\,f^2)-2\,b\,c\,(9\,d^2\,e^2-14\,c\,d\,e\,f+4\,c^2\,f^2)\right)\right.$$

$$\left.\sqrt{e+f\,x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{c\,f}{d\,e}\right]\right)\Big/$$

$$\left(15\,c^{5/2}\,(b\,c-a\,d)^2\,(d\,e-c\,f)^2\,\sqrt{c+d\,x^2}\,\sqrt{\frac{c\,(e+f\,x^2)}{e\,(c+d\,x^2)}}\right)+\left(d\,e^{3/2}\,\sqrt{f}\right.$$

$$\left.(b\,c\,(9\,d\,e-11\,c\,f)-2\,a\,d\,(2\,d\,e-3\,c\,f))\,\sqrt{c+d\,x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]\right)\Big/$$

$$\left(15\,c^3\,(b\,c-a\,d)^2\,(d\,e-c\,f)^2\,\sqrt{\frac{e\,(c+d\,x^2)}{c\,(e+f\,x^2)}}\,\sqrt{e+f\,x^2}\right)+$$

$$\frac{b^3\,e^{3/2}\,\sqrt{c+d\,x^2}\,\text{EllipticPi}\left[1-\frac{b\,e}{a\,f},\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]}{a\,c\,(b\,c-a\,d)^3\,\sqrt{f}\,\sqrt{\frac{e\,(c+d\,x^2)}{c\,(e+f\,x^2)}}\,\sqrt{e+f\,x^2}}$$

Result (type 4, 584 leaves) :

$$\frac{1}{15\,a\,c^3\,\sqrt{\frac{d}{c}}\,(b\,c-a\,d)^3\,(d\,e-c\,f)^2\,(c+d\,x^2)^{5/2}\,\sqrt{e+f\,x^2}}$$

$$\left(-a\,d\,\sqrt{\frac{d}{c}}\,x\,(e+f\,x^2)\,\Big(3\,c^2\,(b\,c-a\,d)^2\,(d\,e-c\,f)^2+\right.$$

$$c\,(b\,c-a\,d)\,(-d\,e+c\,f)\,(a\,d\,(4\,d\,e-3\,c\,f)+b\,c\,(-9\,d\,e+8\,c\,f))\,(c+d\,x^2)+$$

$$\big(a\,b\,c\,d\,(-26\,d^2\,e^2+41\,c\,d\,e\,f-11\,c^2\,f^2)+a^2\,d^2\,(8\,d^2\,e^2-13\,c\,d\,e\,f+3\,c^2\,f^2)+$$

$$b^2\,c^2\,(33\,d^2\,e^2-58\,c\,d\,e\,f+23\,c^2\,f^2)\big)\,(c+d\,x^2)^2\Big)-\mathrm{i}\,(c+d\,x^2)^2\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}$$

$$\Big(a\,d\,e\,\big(a\,b\,c\,d\,(-26\,d^2\,e^2+41\,c\,d\,e\,f-11\,c^2\,f^2)+a^2\,d^2\,(8\,d^2\,e^2-13\,c\,d\,e\,f+3\,c^2\,f^2)+$$

$$b^2\,c^2\,(33\,d^2\,e^2-58\,c\,d\,e\,f+23\,c^2\,f^2)\big)\,\text{EllipticE}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{\frac{d}{c}}\,x\Big],\frac{c\,f}{d\,e}\Big]-$$

$$(d\,e-c\,f)\,\Big(-a\,\big(2\,a\,b\,c\,d^2\,e\,(13\,d\,e-14\,c\,f)+a^2\,d^3\,e\,(-8\,d\,e+9\,c\,f)+$$

$$b^2\,c^2\,(-33\,d^2\,e^2+49\,c\,d\,e\,f-15\,c^2\,f^2)\big)\,\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{\frac{d}{c}}\,x\Big],\frac{c\,f}{d\,e}\Big]+$$

$$\left.15\,b^2\,c^3\,(b\,e-a\,f)\,(-d\,e+c\,f)\,\text{EllipticPi}\Big[\frac{b\,c}{a\,d},\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{\frac{d}{c}}\,x\Big],\frac{c\,f}{d\,e}\Big]\Big)\Big)\right)$$

Problem 71: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(c+d\,x^2)^{3/2}\,(e+f\,x^2)^{3/2}}{a+b\,x^2}\,dx$$

Optimal (type 4, 659 leaves, 14 steps):

$$\frac{(b c-a d)^2 f^2 x \sqrt{c+d x^2}}{b^3 d \sqrt{e+f x^2}}+\frac{2 (b c-a d) f (2 d e-c f) x \sqrt{c+d x^2}}{3 b^2 d \sqrt{e+f x^2}}+$$

$$\frac{\left(3 d^2 e^2+7 c d e f-2 c^2 f^2\right) x \sqrt{c+d x^2}}{15 b d \sqrt{e+f x^2}}+\frac{(b c-a d) f x \sqrt{c+d x^2} \sqrt{e+f x^2}}{3 b^2}+$$

$$\frac{2 (3 d e-c f) x \sqrt{c+d x^2} \sqrt{e+f x^2}}{15 b}+\frac{f x \left(c+d x^2\right)^{3/2} \sqrt{e+f x^2}}{5 b}-$$

$$\left(\sqrt{e} \left(15 a^2 d^2 f^2-20 a b d f (d e+c f)+3 b^2 \left(d^2 e^2+9 c d e f+c^2 f^2\right)\right) \sqrt{c+d x^2}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]\right) \Big/ \left(15 b^3 d \sqrt{f} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right)+$$

$$\left(e^{3/2} \left(15 a^2 d^2 f+3 b^2 c (8 d e+3 c f)-5 a b d (3 d e+5 c f)\right) \sqrt{c+d x^2}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]\right) \Big/ \left(15 b^3 c \sqrt{f} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right)+$$

$$\left((b c-a d)^2 e^{3/2} (b e-a f) \sqrt{c+d x^2} \text{EllipticPi}\left[1-\frac{b e}{a f}, \text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]\right) \Big/$$

$$\left(a b^3 c \sqrt{f} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right)$$

Result (type 4, 445 leaves) :

$$\frac{1}{15\,a\,b^4\sqrt{\frac{d}{c}}\,f\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}$$

$$\left(-\mathrm{i}\,a\,b\,e\,\left(15\,a^2\,d^2\,f^2-20\,a\,b\,d\,f\,\left(d\,e+c\,f\right)+3\,b^2\,\left(d^2\,e^2+9\,c\,d\,e\,f+c^2\,f^2\right)\right)\right.$$

$$\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]-$$

$$\mathrm{i}\,a\,\left(-15\,a^3\,d^2\,f^3+15\,a^2\,b\,d\,f^2\,\left(d\,e+2\,c\,f\right)-3\,b^3\,e\,\left(d^2\,e^2+c\,d\,e\,f-7\,c^2\,f^2\right)+5\,a\,b^2\,f\right.$$

$$\left.\left(d^2\,e^2-7\,c\,d\,e\,f-3\,c^2\,f^2\right)\right)\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]+$$

$$f\left(a\,b^2\sqrt{\frac{d}{c}}\,x\,\left(c+d\,x^2\right)\,\left(e+f\,x^2\right)\,\left(-5\,a\,d\,f+3\,b\,\left(2\,d\,e+2\,c\,f+d\,f\,x^2\right)\right)-15\,\mathrm{i}\,\left(b\,c-a\,d\right)^2\right.$$

$$\left.\left.\left(b\,e-a\,f\right)^2\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)\right)$$

Problem 72: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{c+d\,x^2}\,\left(e+f\,x^2\right)^{3/2}}{a+b\,x^2}\,dx$$

Optimal (type 4, 403 leaves, 7 steps):

$$\frac{f\,(4\,b\,d\,e+b\,c\,f-3\,a\,d\,f)\,x\,\sqrt{c+d\,x^2}}{3\,b^2\,d\,\sqrt{e+f\,x^2}}+\frac{f\,x\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}{3\,b}-$$
$$\left(\sqrt{e}\,\sqrt{f}\,(4\,b\,d\,e+b\,c\,f-3\,a\,d\,f)\,\sqrt{c+d\,x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]\right)\Big/$$
$$\left(3\,b^2\,d\,\sqrt{\frac{e\,(c+d\,x^2)}{c\,(e+f\,x^2)}}\,\sqrt{e+f\,x^2}\right)+$$
$$\frac{\sqrt{e}\,\sqrt{f}\,(5\,b\,e-3\,a\,f)\,\sqrt{c+d\,x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]}{3\,b^2\,\sqrt{\frac{e\,(c+d\,x^2)}{c\,(e+f\,x^2)}}\,\sqrt{e+f\,x^2}}+$$
$$\frac{c^{3/2}\,(b\,e-a\,f)^2\,\sqrt{e+f\,x^2}\,\text{EllipticPi}\left[1-\frac{b\,c}{a\,d},\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{c\,f}{d\,e}\right]}{a\,b^2\,\sqrt{d}\,e\,\sqrt{c+d\,x^2}\,\sqrt{\frac{c\,(e+f\,x^2)}{e\,(c+d\,x^2)}}}$$

Result (type 4, 739 leaves) :

$$\frac{1}{3\,a\,b^3\,\sqrt{\frac{d}{c}}\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}\left(a\,b^2\,c\,\sqrt{\frac{d}{c}}\,e\,f\,x+a\,b^2\,d\,\sqrt{\frac{d}{c}}\,e\,f\,x^3+a\,b^2\,c\,\sqrt{\frac{d}{c}}\,f^2\,x^3+a\,b^2\,d\,\sqrt{\frac{d}{c}}\,f^2\,x^5-\right.$$

$$i\,a\,b\,e\,(4\,b\,d\,e+b\,c\,f-3\,a\,d\,f)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]-$$

$$i\,a\,\left(3\,a^2\,d\,f^2-3\,a\,b\,f\,(d\,e+c\,f)+b^2\,e\,(-d\,e+4\,c\,f)\right)$$

$$\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]-$$

$$3\,i\,b^3\,c\,e^2\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]+$$

$$3\,i\,a\,b^2\,d\,e^2\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]+$$

$$6\,i\,a\,b^2\,c\,e\,f\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]-$$

$$6\,i\,a^2\,b\,d\,e\,f\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]-$$

$$3\,i\,a^2\,b\,c\,f^2\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]+$$

$$\left.3\,i\,a^3\,d\,f^2\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)$$

Problem 73: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(e+f\,x^2\right)^{3/2}}{\left(a+b\,x^2\right)\,\sqrt{c+d\,x^2}}\,dx$$

Optimal (type 4, 328 leaves, 6 steps):

$$\frac{f^2 x \sqrt{c+d x^2}}{b d \sqrt{e+f x^2}}-\frac{\sqrt{e}\, f^{3/2} \sqrt{c+d x^2}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]}{b d \sqrt{\frac{e (c+d x^2)}{c (e+f x^2)}} \sqrt{e+f x^2}}+$$

$$\frac{e^{3/2} \sqrt{f} \sqrt{c+d x^2}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]}{b c \sqrt{\frac{e (c+d x^2)}{c (e+f x^2)}} \sqrt{e+f x^2}}+$$

$$\frac{e^{3/2} (b e-a f) \sqrt{c+d x^2}\, \text{EllipticPi}\left[1-\frac{b e}{a f}, \text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]}{a b c \sqrt{f} \sqrt{\frac{e (c+d x^2)}{c (e+f x^2)}} \sqrt{e+f x^2}}$$

Result (type 4, 184 leaves):

$$-\left(\left(\left(i \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \left(a b e f\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right], \frac{c f}{d e}\right]+\right.\right.\right.\right.$$

$$(b e-a f) \left(a f\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right], \frac{c f}{d e}\right]+(b e-a f)\right.$$

$$\left.\left.\left.\left.\left.\text{EllipticPi}\left[\frac{b c}{a d}, i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right], \frac{c f}{d e}\right]\right)\right)\right)\Big/\left(a b^2 \sqrt{\frac{d}{c}} \sqrt{c+d x^2} \sqrt{e+f x^2}\right)\right)$$

Problem 74: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(e+f x^2\right)^{3/2}}{\left(a+b x^2\right) \left(c+d x^2\right)^{3/2}}\, dx$$

Optimal (type 4, 224 leaves, 3 steps):

$$-\frac{(d e-c f) \sqrt{e+f x^2}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], 1-\frac{c f}{d e}\right]}{\sqrt{c} \sqrt{d} (b c-a d) \sqrt{c+d x^2} \sqrt{\frac{c (e+f x^2)}{e (c+d x^2)}}}+$$

$$\frac{e^{3/2} (b e-a f) \sqrt{c+d x^2}\, \text{EllipticPi}\left[1-\frac{b e}{a f}, \text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]}{a c (b c-a d) \sqrt{f} \sqrt{\frac{e (c+d x^2)}{c (e+f x^2)}} \sqrt{e+f x^2}}$$

Result (type 4, 492 leaves):

$$\frac{1}{a\,b\,d\,(-b\,c+a\,d)\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}$$
$$\sqrt{\frac{d}{c}}\,\Bigg(a\,b\,d\,\sqrt{\frac{d}{c}}\,e^2\,x-a\,b\,c\,\sqrt{\frac{d}{c}}\,e\,f\,x+a\,b\,d\,\sqrt{\frac{d}{c}}\,e\,f\,x^3-a\,b\,c\,\sqrt{\frac{d}{c}}\,f^2\,x^3-$$
$$\mathrm{i}\,a\,b\,e\,(-d\,e+c\,f)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticE}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{\frac{d}{c}}\,x\Big],\frac{c\,f}{d\,e}\Big]+$$
$$\mathrm{i}\,a\,\big(-a\,c\,f^2+b\,e\,(-d\,e+2\,c\,f)\big)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{\frac{d}{c}}\,x\Big],\frac{c\,f}{d\,e}\Big]+$$
$$\mathrm{i}\,b^2\,c\,e^2\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\Big[\frac{b\,c}{a\,d},\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{\frac{d}{c}}\,x\Big],\frac{c\,f}{d\,e}\Big]-$$
$$2\,\mathrm{i}\,a\,b\,c\,e\,f\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\Big[\frac{b\,c}{a\,d},\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{\frac{d}{c}}\,x\Big],\frac{c\,f}{d\,e}\Big]+$$
$$\mathrm{i}\,a^2\,c\,f^2\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\Big[\frac{b\,c}{a\,d},\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{\frac{d}{c}}\,x\Big],\frac{c\,f}{d\,e}\Big]\Bigg)$$

Problem 75: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{\left(e+f\,x^2\right)^{3/2}}{\left(a+b\,x^2\right)\,\left(c+d\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 391 leaves, 6 steps):

$$-\frac{(d e-c f)\, x \sqrt{e+f x^2}}{3 c\, (b c-a d)\, (c+d x^2)^{3/2}}-$$

$$\left(\left(b c\, (5 d e-c f)-2 a d\, (d e+c f)\right) \sqrt{e+f x^2}\ \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], 1-\frac{c f}{d e}\right]\right) \Big/$$

$$\left(3 c^{3/2} \sqrt{d}\, (b c-a d)^2 \sqrt{c+d x^2} \sqrt{\frac{c\, (e+f x^2)}{e\, (c+d x^2)}}\right)+$$

$$\frac{e^{3/2} \sqrt{f} \sqrt{c+d x^2}\ \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]}{3 c^2\, (b c-a d) \sqrt{\frac{e\, (c+d x^2)}{c\, (e+f x^2)}} \sqrt{e+f x^2}}+$$

$$\left(b e^{3/2}\, (b e-a f) \sqrt{c+d x^2}\ \text{EllipticPi}\left[1-\frac{b e}{a f}, \text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]\right) \Big/$$

$$\left(a c\, (b c-a d)^2 \sqrt{f} \sqrt{\frac{e\, (c+d x^2)}{c\, (e+f x^2)}} \sqrt{e+f x^2}\right)$$

Result (type 4, 999 leaves):

$$\frac{1}{3\,a\,c^2\,\sqrt{\frac{d}{c}}\,\left(b\,c-a\,d\right)^2\,\left(c+d\,x^2\right)^{3/2}\,\sqrt{e+f\,x^2}}$$

$$\left(3\,a^2\,c\,d^2\,\sqrt{\frac{d}{c}}\,e^2\,x-6\,a\,b\,c^3\,\left(\frac{d}{c}\right)^{3/2}\,e^2\,x+2\,a\,b\,c^3\,\sqrt{\frac{d}{c}}\,e\,f\,x+a^2\,c^3\,\left(\frac{d}{c}\right)^{3/2}\,e\,f\,x-\right.$$

$$5\,a\,b\,c\,d^2\,\sqrt{\frac{d}{c}}\,e^2\,x^3+2\,a^2\,d^3\,\sqrt{\frac{d}{c}}\,e^2\,x^3+5\,a^2\,c\,d^2\,\sqrt{\frac{d}{c}}\,e\,f\,x^3-5\,a\,b\,c^3\,\left(\frac{d}{c}\right)^{3/2}\,e\,f\,x^3+$$

$$2\,a\,b\,c^3\,\sqrt{\frac{d}{c}}\,f^2\,x^3+a^2\,c^3\,\left(\frac{d}{c}\right)^{3/2}\,f^2\,x^3-5\,a\,b\,c\,d^2\,\sqrt{\frac{d}{c}}\,e\,f\,x^5+2\,a^2\,d^3\,\sqrt{\frac{d}{c}}\,e\,f\,x^5+$$

$$2\,a^2\,c\,d^2\,\sqrt{\frac{d}{c}}\,f^2\,x^5+a\,b\,c^3\,\left(\frac{d}{c}\right)^{3/2}\,f^2\,x^5+\mathrm{i}\,a\,e\,\left(b\,c\,\left(-5\,d\,e+c\,f\right)+2\,a\,d\,\left(d\,e+c\,f\right)\right)$$

$$\left(c+d\,x^2\right)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]-\mathrm{i}\,a\,\left(-d\,e+c\,f\right)$$

$$\left(5\,b\,c\,e-2\,a\,d\,e-3\,a\,c\,f\right)\,\left(c+d\,x^2\right)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]-$$

$$3\,\mathrm{i}\,b^2\,c^3\,e^2\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]+$$

$$6\,\mathrm{i}\,a\,b\,c^3\,e\,f\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]-$$

$$3\,\mathrm{i}\,a^2\,c^3\,f^2\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]-$$

$$3\,\mathrm{i}\,b^2\,c^2\,d\,e^2\,x^2\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]+$$

$$6\,\mathrm{i}\,a\,b\,c^2\,d\,e\,f\,x^2\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]-$$

$$\left.3\,\mathrm{i}\,a^2\,c^2\,d\,f^2\,x^2\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]\right)$$

Problem 76: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(e+f x^2\right)^{3/2}}{\left(a+b x^2\right) \left(c+d x^2\right)^{7/2}} \, dx$$

Optimal (type 4, 639 leaves, 9 steps):

$$-\frac{(d e-c f) x \sqrt{e+f x^2}}{5 c (b c-a d) \left(c+d x^2\right)^{5/2}}-\frac{(3 b c (3 d e-c f)-2 a d (2 d e+c f)) x \sqrt{e+f x^2}}{15 c^2 (b c-a d)^2 \left(c+d x^2\right)^{3/2}}-\frac{b \sqrt{d} (b e-a f) \sqrt{e+f x^2} \, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1-\frac{c f}{d e}\right]}{\sqrt{c} (b c-a d)^3 \sqrt{c+d x^2} \sqrt{\frac{c \left(e+f x^2\right)}{e \left(c+d x^2\right)}}}+$$

$$\left(\left(a d \left(8 d^2 e^2-3 c d e f-2 c^2 f^2\right)-3 b c \left(6 d^2 e^2-6 c d e f+c^2 f^2\right)\right) \sqrt{e+f x^2} \, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1-\frac{c f}{d e}\right]\right) \Big/ \left(15 c^{5/2} \sqrt{d} (b c-a d)^2 (d e-c f) \sqrt{c+d x^2} \sqrt{\frac{c \left(e+f x^2\right)}{e \left(c+d x^2\right)}}\right)+$$

$$\left(e^{3/2} \sqrt{f} (3 b c (3 d e-2 c f)-a d (4 d e-c f)) \sqrt{c+d x^2} \, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]\right) \Big/ \left(15 c^3 (b c-a d)^2 (d e-c f) \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right)+$$

$$\left(b^2 e^{3/2} (b e-a f) \sqrt{c+d x^2} \, \text{EllipticPi}\left[1-\frac{b e}{a f}, \text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]\right) \Big/ \left(a c (b c-a d)^3 \sqrt{f} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right)$$

Result (type 4, 570 leaves):

$$\frac{1}{15\,a\,c^3\,\sqrt{\frac{d}{c}}\,(b\,c-a\,d)^3\,(d\,e-c\,f)\,(c+d\,x^2)^{5/2}\,\sqrt{e+f\,x^2}}$$

$$\Bigg(-a\,\sqrt{\frac{d}{c}}\,x\,(e+f\,x^2)\,\Big(3\,c^2\,(b\,c-a\,d)^2\,(d\,e-c\,f)^2+$$
$$c\,(b\,c-a\,d)\,(-d\,e+c\,f)\,(3\,b\,c\,(-3\,d\,e+c\,f)+2\,a\,d\,(2\,d\,e+c\,f))\,(c+d\,x^2)+$$
$$\big(a^2\,d^2\,(8\,d^2\,e^2-3\,c\,d\,e\,f-2\,c^2\,f^2)+3\,b^2\,c^2\,(11\,d^2\,e^2-11\,c\,d\,e\,f+c^2\,f^2)+$$
$$2\,a\,b\,c\,d\,(-13\,d^2\,e^2+3\,c\,d\,e\,f+7\,c^2\,f^2)\big)\,(c+d\,x^2)^2\Big)+i\,(c+d\,x^2)^2\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}$$
$$\Big(a\,e\,\big(-3\,b^2\,c^2\,(11\,d^2\,e^2-11\,c\,d\,e\,f+c^2\,f^2)+a^2\,d^2\,(-8\,d^2\,e^2+3\,c\,d\,e\,f+2\,c^2\,f^2)-$$
$$2\,a\,b\,c\,d\,(-13\,d^2\,e^2+3\,c\,d\,e\,f+7\,c^2\,f^2)\big)\,\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\sqrt{\frac{d}{c}}\,x\Big],\frac{c\,f}{d\,e}\Big]+(d\,e-c\,f)$$
$$\Big(a\,\big(3\,b^2\,c^2\,e\,(11\,d\,e-8\,c\,f)+a^2\,d^2\,e\,(8\,d\,e+c\,f)+a\,b\,c\,(-26\,d^2\,e^2-7\,c\,d\,e\,f+15\,c^2\,f^2)\big)$$
$$\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\sqrt{\frac{d}{c}}\,x\Big],\frac{c\,f}{d\,e}\Big]-$$
$$15\,b\,c^3\,(b\,e-a\,f)^2\,\text{EllipticPi}\Big[\frac{b\,c}{a\,d},i\,\text{ArcSinh}\Big[\sqrt{\frac{d}{c}}\,x\Big],\frac{c\,f}{d\,e}\Big]\Big)\Big)\Bigg)$$

Problem 77: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(c+d\,x^2)^{5/2}}{(a+b\,x^2)\,\sqrt{e+f\,x^2}}\,dx$$

Optimal (type 4, 621 leaves, 12 steps):

$$\frac{d\,(b c-a d)\,x\sqrt{c+d x^2}}{b^2\sqrt{e+f x^2}}-\frac{2 d\,(d e-2 c f)\,x\sqrt{c+d x^2}}{3 b f\sqrt{e+f x^2}}+\frac{d^2 x\sqrt{c+d x^2}\,\sqrt{e+f x^2}}{3 b f}-$$

$$\frac{d\,(b c-a d)\sqrt{e}\,\sqrt{c+d x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{b^2\sqrt{f}\,\sqrt{\frac{e\,(c+d x^2)}{c\,(e+f x^2)}}\,\sqrt{e+f x^2}}+$$

$$\frac{2 d\sqrt{e}\,(d e-2 c f)\sqrt{c+d x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{3 b f^{3/2}\sqrt{\frac{e\,(c+d x^2)}{c\,(e+f x^2)}}\,\sqrt{e+f x^2}}+$$

$$\frac{d\,(b c-a d)\sqrt{e}\,\sqrt{c+d x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{b^2\sqrt{f}\,\sqrt{\frac{e\,(c+d x^2)}{c\,(e+f x^2)}}\,\sqrt{e+f x^2}}-$$

$$\frac{d\sqrt{e}\,(d e-3 c f)\sqrt{c+d x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{3 b f^{3/2}\sqrt{\frac{e\,(c+d x^2)}{c\,(e+f x^2)}}\,\sqrt{e+f x^2}}+$$

$$\frac{c^{3/2}\,(b c-a d)^2\sqrt{e+f x^2}\,\text{EllipticPi}\left[1-\frac{b c}{a d},\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{c f}{d e}\right]}{a b^2\sqrt{d}\;e\sqrt{c+d x^2}\,\sqrt{\frac{c\,(e+f x^2)}{e\,(c+d x^2)}}}$$

Result (type 4, 350 leaves):

$$\frac{1}{3 a b^3\sqrt{\frac{d}{c}}\,f^2\sqrt{c+d x^2}\,\sqrt{e+f x^2}}$$

$$\left(-i a b d^2 e\,(-2 b d e+7 b c f-3 a d f)\sqrt{1+\frac{d x^2}{c}}\,\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c f}{d e}\right]-\right.$$

$$i a d\left(3 a^2 d^2 f^2+3 a b d f\,(d e-3 c f)+b^2\left(2 d^2 e^2-8 c d e f+9 c^2 f^2\right)\right)\sqrt{1+\frac{d x^2}{c}}$$

$$\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c f}{d e}\right]+f\left(a b^2 c d\left(\frac{d}{c}\right)^{3/2}x\,(c+d x^2)\,(e+f x^2)-\right.$$

$$\left.\left.3 i\,(b c-a d)^3 f\sqrt{1+\frac{d x^2}{c}}\,\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticPi}\left[\frac{b c}{a d},i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c f}{d e}\right]\right)\right)$$

Problem 78: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(c+d x^2\right)^{3/2}}{\left(a+b x^2\right) \sqrt{e+f x^2}} \, dx$$

Optimal (type 4, 319 leaves, 6 steps):

$$\frac{d x \sqrt{c+d x^2}}{b \sqrt{e+f x^2}}-\frac{d \sqrt{e} \sqrt{c+d x^2} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]}{b \sqrt{f} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}}+$$
$$\frac{d \sqrt{e} \sqrt{c+d x^2} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]}{b \sqrt{f} \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}}+$$
$$\frac{c^{3/2} (b c-a d) \sqrt{e+f x^2} \text{EllipticPi}\left[1-\frac{b c}{a d}, \text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1-\frac{c f}{d e}\right]}{a b \sqrt{d} e \sqrt{c+d x^2} \sqrt{\frac{c \left(e+f x^2\right)}{e \left(c+d x^2\right)}}}$$

Result (type 4, 197 leaves):

$$-\left(\left(i \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \left(a b d^2 e \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{d}{c}} x\right], \frac{c f}{d e}\right]-\right.\right.\right.$$
$$a d (b d e-2 b c f+a d f) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{d}{c}} x\right], \frac{c f}{d e}\right]+(b c-a d)^2 f$$
$$\left.\left.\left.\text{EllipticPi}\left[\frac{b c}{a d}, i \text{ArcSinh}\left[\sqrt{\frac{d}{c}} x\right], \frac{c f}{d e}\right]\right)\right) \Big/ \left(a b^2 \sqrt{\frac{d}{c}} f \sqrt{c+d x^2} \sqrt{e+f x^2}\right)\right)$$

Problem 79: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{c+d x^2}}{\left(a+b x^2\right) \sqrt{e+f x^2}} \, dx$$

Optimal (type 4, 102 leaves, 1 step):

$$\frac{c^{3/2} \sqrt{e+f x^2} \text{EllipticPi}\left[1-\frac{b c}{a d}, \text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1-\frac{c f}{d e}\right]}{a \sqrt{d} e \sqrt{c+d x^2} \sqrt{\frac{c \left(e+f x^2\right)}{e \left(c+d x^2\right)}}}$$

Result (type 4, 143 leaves):

$$-\left(\left(\left(i \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \left(a d \,\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right], \frac{c f}{d e}\right] + (b c - a d)\, \text{EllipticPi}\left[\frac{b c}{a d}, i \,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right], \frac{c f}{d e}\right]\right)\right)\right) \Big/ \left(a b \sqrt{\frac{d}{c}} \sqrt{c+d x^2} \sqrt{e+f x^2}\right)\right)$$

Problem 80: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+b x^2\right) \sqrt{c+d x^2} \sqrt{e+f x^2}}\, dx$$

Optimal (type 4, 100 leaves, 3 steps):

$$\frac{\sqrt{-c} \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \,\text{EllipticPi}\left[\frac{b c}{a d}, \text{ArcSin}\left[\frac{\sqrt{d}\, x}{\sqrt{-c}}\right], \frac{c f}{d e}\right]}{a \sqrt{d} \sqrt{c+d x^2} \sqrt{e+f x^2}}$$

Result (type 4, 101 leaves):

$$-\frac{i \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \,\text{EllipticPi}\left[\frac{b c}{a d}, i \,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right], \frac{c f}{d e}\right]}{a \sqrt{\frac{d}{c}} \sqrt{c+d x^2} \sqrt{e+f x^2}}$$

Problem 81: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+b x^2\right) \left(c+d x^2\right)^{3/2} \sqrt{e+f x^2}}\, dx$$

Optimal (type 4, 344 leaves, 5 steps):

$$-\frac{d^{3/2} \sqrt{e+f x^2} \,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], 1-\frac{c f}{d e}\right]}{\sqrt{c}\, (b c - a d) (d e - c f) \sqrt{c+d x^2} \sqrt{\frac{c (e+f x^2)}{e (c+d x^2)}}} - \left(d \sqrt{e}\, (b d e - 2 b c f + a d f) \sqrt{c+d x^2} \,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]\right) \Big/ \left(c\, (b c - a d)^2 \sqrt{f}\, (d e - c f) \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right) + \frac{b^2 c^{3/2} \sqrt{e+f x^2} \,\text{EllipticPi}\left[1-\frac{b c}{a d}, \text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], 1-\frac{c f}{d e}\right]}{a \sqrt{d}\, (b c - a d)^2 e \sqrt{c+d x^2} \sqrt{\frac{c (e+f x^2)}{e (c+d x^2)}}}$$

Result (type 4, 365 leaves):

$$\frac{1}{a\,d\,(-b\,c+a\,d)\,(d\,e-c\,f)\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}\sqrt{\frac{d}{c}}\left(a\,c\,d\left(\frac{d}{c}\right)^{3/2}e\,x+a\,c\,d\left(\frac{d}{c}\right)^{3/2}f\,x^3+i\,a\,d^2\,e\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]+i\,a\,d\,(-d\,e+c\,f)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]+i\,b\,c\,d\,e\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]-i\,b\,c^2\,f\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)$$

Problem 82: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{(a+b\,x^2)\,(c+d\,x^2)^{5/2}\,\sqrt{e+f\,x^2}}\,dx$$

Optimal (type 4, 435 leaves, 8 steps):

$$-\frac{d^2\,x\,\sqrt{e+f\,x^2}}{3\,c\,(b\,c-a\,d)\,(d\,e-c\,f)\,(c+d\,x^2)^{3/2}}-\left(d^{3/2}\,(b\,c\,(5\,d\,e-7\,c\,f)-2\,a\,d\,(d\,e-2\,c\,f))\,\sqrt{e+f\,x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{c\,f}{d\,e}\right]\right)\Bigg/\left(3\,c^{3/2}\,(b\,c-a\,d)^2\,(d\,e-c\,f)^2\,\sqrt{c+d\,x^2}\,\sqrt{\frac{c\,(e+f\,x^2)}{e\,(c+d\,x^2)}}\right)-\left(d\,\sqrt{e}\,\sqrt{f}\,(a\,d\,(d\,e-3\,c\,f)-2\,b\,c\,(2\,d\,e-3\,c\,f))\,\sqrt{c+d\,x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]\right)\Bigg/\left(3\,c^2\,(b\,c-a\,d)^2\,(d\,e-c\,f)^2\,\sqrt{\frac{e\,(c+d\,x^2)}{c\,(e+f\,x^2)}}\,\sqrt{e+f\,x^2}\right)+\frac{b^2\,\sqrt{-c}\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},\text{ArcSin}\left[\frac{\sqrt{d}\,x}{\sqrt{-c}}\right],\frac{c\,f}{d\,e}\right]}{a\,\sqrt{d}\,(b\,c-a\,d)^2\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}$$

Result (type 4, 433 leaves):

$$\frac{1}{3\,a\,c^2\,\sqrt{\frac{d}{c}}\,\left(b\,c-a\,d\right)^2\,\left(d\,e-c\,f\right)^2\,\left(c+d\,x^2\right)^{3/2}\,\sqrt{e+f\,x^2}}\left(a\,c\,d\,\left(\frac{d}{c}\right)^{3/2}\,x\,\left(e+f\,x^2\right)\right.$$

$$\left(b\,c\,\left(-6\,c\,d\,e+8\,c^2\,f-5\,d^2\,e\,x^2+7\,c\,d\,f\,x^2\right)+a\,d\,\left(-5\,c^2\,f+2\,d^2\,e\,x^2+c\,d\,\left(3\,e-4\,f\,x^2\right)\right)\right)+$$

$$i\,a\,d^2\,e\,\left(2\,a\,d\,\left(d\,e-2\,c\,f\right)+b\,c\,\left(-5\,d\,e+7\,c\,f\right)\right)\,\left(c+d\,x^2\right)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]+i\,a\,d\,\left(-d\,e+c\,f\right)\,\left(a\,d\,\left(2\,d\,e-3\,c\,f\right)+b\,c\,\left(-5\,d\,e+6\,c\,f\right)\right)$$

$$\left(c+d\,x^2\right)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]-$$

$$\left.3\,i\,b^2\,c^2\,\left(d\,e-c\,f\right)^2\,\left(c+d\,x^2\right)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},\,i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c\,f}{d\,e}\right]\right)$$

Problem 83: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d\,x^2\right)^{5/2}}{\left(a+b\,x^2\right)\,\left(e+f\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 980 leaves, 14 steps):

$$\frac{(b c-a d)\,(b d e+4 b c f-3 a d f)\,x\,\sqrt{c+d x^2}}{3 b\,(b e-a f)^2\,\sqrt{e+f x^2}}+$$

$$\left(\left(b e\,(6 d^2 e^2-7 c d e f-c^2 f^2)-a f\,(8 d^2 e^2-13 c d e f+3 c^2 f^2)\right)x\,\sqrt{c+d x^2}\right)\Big/$$

$$\left(3 e f\,(b e-a f)^2\,\sqrt{e+f x^2}\right)+\frac{(d e-c f)\,x\,(c+d x^2)^{3/2}}{e\,(b e-a f)\,\sqrt{e+f x^2}}+\frac{d\,(b c-a d)\,x\,\sqrt{c+d x^2}\,\sqrt{e+f x^2}}{3\,(b e-a f)^2}+$$

$$\frac{d\,\left(a f\,(4 d e-3 c f)-b e\,(3 d e-2 c f)\right)x\,\sqrt{c+d x^2}\,\sqrt{e+f x^2}}{3 e f\,(b e-a f)^2}-$$

$$\left((b c-a d)\,\sqrt{e}\,(b d e+4 b c f-3 a d f)\,\sqrt{c+d x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]\right)\Big/$$

$$\left(3 b\,\sqrt{f}\,(b e-a f)^2\,\sqrt{\frac{e\,(c+d x^2)}{c\,(e+f x^2)}}\,\sqrt{e+f x^2}\right)-$$

$$\left(\left(b e\,(6 d^2 e^2-7 c d e f-c^2 f^2)-a f\,(8 d^2 e^2-13 c d e f+3 c^2 f^2)\right)\sqrt{c+d x^2}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]\right)\Big/\left(3\,\sqrt{e}\,f^{3/2}\,(b e-a f)^2\,\sqrt{\frac{e\,(c+d x^2)}{c\,(e+f x^2)}}\,\sqrt{e+f x^2}\right)+$$

$$\left(d\,(5 b c-3 a d)\,(b c-a d)\,e^{3/2}\,\sqrt{c+d x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]\right)\Big/$$

$$\left(3 b c\,\sqrt{f}\,(b e-a f)^2\,\sqrt{\frac{e\,(c+d x^2)}{c\,(e+f x^2)}}\,\sqrt{e+f x^2}\right)-$$

$$\left(\sqrt{e}\,\left(2 a d f\,(2 d e-3 c f)-b\,(3 d^2 e^2-2 c d e f-3 c^2 f^2)\right)\sqrt{c+d x^2}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]\right)\Big/\left(3 f^{3/2}\,(b e-a f)^2\,\sqrt{\frac{e\,(c+d x^2)}{c\,(e+f x^2)}}\,\sqrt{e+f x^2}\right)+$$

$$\frac{(b c-a d)^3\,e^{3/2}\,\sqrt{c+d x^2}\,\text{EllipticPi}\left[1-\frac{b e}{a f},\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{a b c\,\sqrt{f}\,(b e-a f)^2\,\sqrt{\frac{e\,(c+d x^2)}{c\,(e+f x^2)}}\,\sqrt{e+f x^2}}$$

Result (type 4, 352 leaves) :

$$\frac{1}{a\, b^2 \sqrt{\frac{d}{c}}\; e\, f^2\, (b\, e - a\, f)\, \sqrt{c + d\, x^2}\, \sqrt{e + f\, x^2}} \left(-\mathrm{i}\, a\, b\, d\, e\, \left(-a\, d^2\, e\, f + b\, \left(2\, d^2\, e^2 - 2\, c\, d\, e\, f + c^2\, f^2\right)\right) \sqrt{1 + \frac{d\, x^2}{c}}\, \sqrt{1 + \frac{f\, x^2}{e}}\right.$$

$$\text{EllipticE}\left[\mathrm{i}\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right], \frac{c\, f}{d\, e}\right] - \mathrm{i}\, a\, d^2\, e\, (b\, e - a\, f)\, (-2\, b\, d\, e + 3\, b\, c\, f - a\, d\, f)\, \sqrt{1 + \frac{d\, x^2}{c}}$$

$$\sqrt{1 + \frac{f\, x^2}{e}}\; \text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right], \frac{c\, f}{d\, e}\right] - f \left(a\, b^2 \sqrt{\frac{d}{c}}\, (d\, e - c\, f)^2\, x\, \left(c + d\, x^2\right) + \right.$$

$$\left.\left. \mathrm{i}\, (b\, c - a\, d)^3\, e\, f\, \sqrt{1 + \frac{d\, x^2}{c}}\, \sqrt{1 + \frac{f\, x^2}{e}}\; \text{EllipticPi}\left[\frac{b\, c}{a\, d}, \mathrm{i}\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right], \frac{c\, f}{d\, e}\right]\right)\right)$$

Problem 84: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(c + d\, x^2\right)^{3/2}}{\left(a + b\, x^2\right)\, \left(e + f\, x^2\right)^{3/2}}\, dx$$

Optimal (type 4, 223 leaves, 3 steps):

$$\frac{(d\, e - c\, f)\, \sqrt{c + d\, x^2}\; \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right], 1 - \frac{d\, e}{c\, f}\right]}{\sqrt{e}\, \sqrt{f}\, (b\, e - a\, f)\, \sqrt{\frac{e\, (c + d\, x^2)}{c\, (e + f\, x^2)}}\, \sqrt{e + f\, x^2}} +$$

$$\frac{c^{3/2}\, (b\, c - a\, d)\, \sqrt{e + f\, x^2}\; \text{EllipticPi}\left[1 - \frac{b\, c}{a\, d}, \text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], 1 - \frac{c\, f}{d\, e}\right]}{a\, \sqrt{d}\, e\, (b\, e - a\, f)\, \sqrt{c + d\, x^2}\, \sqrt{\frac{c\, (e + f\, x^2)}{e\, (c + d\, x^2)}}}$$

Result (type 4, 304 leaves):

$$\left(a\,b\,\sqrt{\frac{d}{c}}\,f\,(d\,e-c\,f)\,x\,(c+d\,x^2)-\right.$$
$$i\,a\,b\,d\,e\,(-d\,e+c\,f)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]-$$
$$i\,a\,d^2\,e\,(b\,e-a\,f)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]-$$
$$\left.i\,(b\,c-a\,d)^2\,e\,f\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)\Big/$$
$$\left(a\,b\,\sqrt{\frac{d}{c}}\,e\,f\,(b\,e-a\,f)\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}\right)$$

Problem 85: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{c+d\,x^2}}{(a+b\,x^2)\,(e+f\,x^2)^{3/2}}\,dx$$

Optimal (type 4, 209 leaves, 3 steps):

$$-\frac{\sqrt{f}\,\sqrt{c+d\,x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]}{\sqrt{e}\,(b\,e-a\,f)\,\sqrt{\frac{e\,(c+d\,x^2)}{c\,(e+f\,x^2)}}\,\sqrt{e+f\,x^2}}+$$
$$\frac{b\,c^{3/2}\,\sqrt{e+f\,x^2}\,\text{EllipticPi}\left[1-\frac{b\,c}{a\,d},\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{c\,f}{d\,e}\right]}{a\,\sqrt{d}\,e\,(b\,e-a\,f)\,\sqrt{c+d\,x^2}\,\sqrt{\frac{c\,(e+f\,x^2)}{e\,(c+d\,x^2)}}}$$

Result (type 4, 207 leaves):

$$\left(-a\,\sqrt{\frac{d}{c}}\,f\,x\,(c+d\,x^2)-i\,a\,d\,e\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]-\right.$$
$$\left.i\,(b\,c-a\,d)\,e\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)\Big/$$
$$\left(a\,\sqrt{\frac{d}{c}}\,e\,(b\,e-a\,f)\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}\right)$$

Problem 86: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+b x^2\right) \sqrt{c+d x^2} \left(e+f x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 344 leaves, 5 steps):

$$\frac{f^{3/2} \sqrt{c+d x^2} \, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]}{\sqrt{e} (b e-a f) (d e-c f) \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}} - \left(\sqrt{e} \sqrt{f} (2 b d e-b c f-a d f) \sqrt{c+d x^2} \, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]\right) \Big/ \left(c (b e-a f)^2 (d e-c f) \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right) + \frac{b^2 e^{3/2} \sqrt{c+d x^2} \, \text{EllipticPi}\left[1-\frac{b e}{a f}, \text{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right], 1-\frac{d e}{c f}\right]}{a c \sqrt{f} (b e-a f)^2 \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}}$$

Result (type 4, 221 leaves):

$$\left(-a \sqrt{\frac{d}{c}} f^2 x \left(c+d x^2\right) - i a d e f \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \, \text{EllipticE}\left[i \, \text{ArcSinh}\left[\sqrt{\frac{d}{c}} x\right], \frac{c f}{d e}\right] - i b e (-d e+c f) \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \, \text{EllipticPi}\left[\frac{b c}{a d}, i \, \text{ArcSinh}\left[\sqrt{\frac{d}{c}} x\right], \frac{c f}{d e}\right]\right) \Big/ \left(a \sqrt{\frac{d}{c}} e (-b e+a f) (d e-c f) \sqrt{c+d x^2} \sqrt{e+f x^2}\right)$$

Problem 87: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b x^2\right) \left(c+d x^2\right)^{3/2} \left(e+f x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 539 leaves, 8 steps):

$$-\frac{d^2 x}{c\,(b c-a d)\,(d e-c f)\,\sqrt{c+d x^2}\,\sqrt{e+f x^2}}-\frac{b^2\sqrt{f}\,\sqrt{c+d x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{(b c-a d)^2\sqrt{e}\,(b e-a f)\,\sqrt{\frac{e\,(c+d x^2)}{c\,(e+f x^2)}}\,\sqrt{e+f x^2}}-$$

$$\left(d\sqrt{f}\,\left(2 b c^2 f-a d\,(d e+c f)\right)\sqrt{c+d x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]\right)\Bigg/$$

$$\left(c\,(b c-a d)^2\sqrt{e}\,(d e-c f)^2\sqrt{\frac{e\,(c+d x^2)}{c\,(e+f x^2)}}\,\sqrt{e+f x^2}\right)-$$

$$\left(d^2\sqrt{e}\,(b d e-3 b c f+2 a d f)\sqrt{c+d x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]\right)\Bigg/$$

$$\left(c\,(b c-a d)^2\sqrt{f}\,(d e-c f)^2\sqrt{\frac{e\,(c+d x^2)}{c\,(e+f x^2)}}\,\sqrt{e+f x^2}\right)+$$

$$\frac{b^3 c^{3/2}\sqrt{e+f x^2}\,\text{EllipticPi}\left[1-\frac{b c}{a d},\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],1-\frac{c f}{d e}\right]}{a\sqrt{d}\,(b c-a d)^2 e\,(b e-a f)\sqrt{c+d x^2}\sqrt{\frac{c\,(e+f x^2)}{e\,(c+d x^2)}}}$$

Result (type 4, 1284 leaves):

$$\sqrt{c+d x^2}\,\sqrt{e+f x^2}\left(-\frac{d^3 x}{c\,(b c-a d)\,(-d e+c f)^2\,(c+d x^2)}-\frac{f^3 x}{e\,(b e-a f)\,(d e-c f)^2\,(e+f x^2)}\right)-$$

$$\frac{1}{c\,(b c-a d)\,e\,(b e-a f)\,(-d e+c f)^2\sqrt{c+d x^2}\,\sqrt{e+f x^2}}$$

$$\sqrt{(c+d x^2)\,(e+f x^2)}\,\Bigg(\Bigg(i\,b\,d^3 e^3\sqrt{1+\frac{d x^2}{c}}\,\sqrt{1+\frac{f x^2}{e}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c f}{d e}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c f}{d e}\right]\right)\Bigg)\Bigg/$$

$$\left(\sqrt{\frac{d}{c}}\,\sqrt{(c+d x^2)\,(e+f x^2)}\right)-\Bigg(i\,a\,d^3 e^2 f\sqrt{1+\frac{d x^2}{c}}\,\sqrt{1+\frac{f x^2}{e}}$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c f}{d e}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c f}{d e}\right]\right)\Bigg)\Bigg/$$

$$\left(\sqrt{\frac{d}{c}}\,\sqrt{(c+d x^2)\,(e+f x^2)}\right)+\Bigg(i\,b\,c^2 d\,e\,f^2\sqrt{1+\frac{d x^2}{c}}\,\sqrt{1+\frac{f x^2}{e}}$$

$$\left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\; x\right], \frac{c f}{d e}\right] - \text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\; x\right], \frac{c f}{d e}\right]\right)\right) \Big/$$

$$\left(\sqrt{\frac{d}{c}}\; \sqrt{\left(c + d x^2\right)\; \left(e + f x^2\right)}\right) - \left(i\, a\, c\, d^2\, e\, f^2 \sqrt{1 + \frac{d x^2}{c}}\; \sqrt{1 + \frac{f x^2}{e}}\right.$$

$$\left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\; x\right], \frac{c f}{d e}\right] - \text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\; x\right], \frac{c f}{d e}\right]\right)\right) \Big/$$

$$\left(\sqrt{\frac{d}{c}}\; \sqrt{\left(c + d x^2\right)\; \left(e + f x^2\right)}\right) + \left(i\, b\, c\, d^2\, e^2\, f \sqrt{1 + \frac{d x^2}{c}}\; \sqrt{1 + \frac{f x^2}{e}}\right.$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\; x\right], \frac{c f}{d e}\right]\right) \Big/ \left(\sqrt{\frac{d}{c}}\; \sqrt{\left(c + d x^2\right)\; \left(e + f x^2\right)}\right) +$$

$$\left(i\, b\, c^2\, d\, e\, f^2 \sqrt{1 + \frac{d x^2}{c}}\; \sqrt{1 + \frac{f x^2}{e}}\; \text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\; x\right], \frac{c f}{d e}\right]\right) \Big/$$

$$\left(\sqrt{\frac{d}{c}}\; \sqrt{\left(c + d x^2\right)\; \left(e + f x^2\right)}\right) -$$

$$\left(2\, i\, a\, c\, d^2\, e\, f^2 \sqrt{1 + \frac{d x^2}{c}}\; \sqrt{1 + \frac{f x^2}{e}}\; \text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\; x\right], \frac{c f}{d e}\right]\right) \Big/$$

$$\left(\sqrt{\frac{d}{c}}\; \sqrt{\left(c + d x^2\right)\; \left(e + f x^2\right)}\right) +$$

$$\left(i\, b^2\, c\, d^2\, e^3 \sqrt{1 + \frac{d x^2}{c}}\; \sqrt{1 + \frac{f x^2}{e}}\; \text{EllipticPi}\left[\frac{b c}{a d}, i \,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\; x\right], \frac{c f}{d e}\right]\right) \Big/$$

$$\left(a \sqrt{\frac{d}{c}}\; \sqrt{\left(c + d x^2\right)\; \left(e + f x^2\right)}\right) -$$

$$\left(2\, i\, b^2\, c^2\, d\, e^2\, f \sqrt{1 + \frac{d x^2}{c}}\; \sqrt{1 + \frac{f x^2}{e}}\; \text{EllipticPi}\left[\frac{b c}{a d}, i \,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\; x\right], \frac{c f}{d e}\right]\right) \Big/$$

$$\left(a \sqrt{\frac{d}{c}}\; \sqrt{\left(c + d x^2\right)\; \left(e + f x^2\right)}\right) +$$

$$\left(i\, b^2\, c^3\, e\, f^2 \sqrt{1+\frac{d\, x^2}{c}} \sqrt{1+\frac{f\, x^2}{e}}\; \text{EllipticPi}\left[\frac{b\, c}{a\, d}, i\, \text{ArcSinh}\left[\sqrt{\frac{d}{c}}\, x\right], \frac{c\, f}{d\, e}\right]\right) \Big/ \left(a \sqrt{\frac{d}{c}} \sqrt{\left(c+d\, x^2\right) \left(e+f\, x^2\right)}\right)\Bigg)$$

Problem 88: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b\, x^2\right) \left(c+d\, x^2\right)^{5/2} \left(e+f\, x^2\right)^{3/2}}\, dx$$

Optimal (type 4, 814 leaves, 11 steps):

$$-\frac{d^2 x}{3 c (b c-a d) (d e-c f) \left(c+d x^2\right)^{3/2} \sqrt{e+f x^2}}-\frac{d^2 (b c (5 d e-9 c f)-2 a d (d e-3 c f)) x}{3 c^2 (b c-a d)^2 (d e-c f)^2 \sqrt{c+d x^2} \sqrt{e+f x^2}}+$$

$$\frac{b^2 f^{3/2} \sqrt{c+d x^2}\ \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{(b c-a d)^2 \sqrt{e}\, (b e-a f) (d e-c f) \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}}-$$

$$\left(d \sqrt{f} \left(b c \left(5 d^2 e^2-7 c d e f-6 c^2 f^2\right)-a d \left(2 d^2 e^2-7 c d e f-3 c^2 f^2\right)\right)\right.$$
$$\left.\sqrt{c+d x^2}\ \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]\right)\Big/$$
$$\left(3 c^2 (b c-a d)^2 \sqrt{e}\, (d e-c f)^3 \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right)-$$

$$\left(b^2 \sqrt{e} \sqrt{f} (2 b d e-b c f-a d f) \sqrt{c+d x^2}\ \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]\right)\Big/$$
$$\left(c (b c-a d)^2 (b e-a f)^2 (d e-c f) \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right)+$$

$$\left(d^2 \sqrt{e} \sqrt{f} (b c (7 d e-15 c f)-a d (d e-9 c f)) \sqrt{c+d x^2}\ \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]\right)\Big/$$
$$\left(3 c^2 (b c-a d)^2 (d e-c f)^3 \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}\right)+$$

$$\frac{b^4 e^{3/2} \sqrt{c+d x^2}\ \text{EllipticPi}\left[1-\frac{b e}{a f},\text{ArcTan}\left[\frac{\sqrt{f}\, x}{\sqrt{e}}\right],1-\frac{d e}{c f}\right]}{a c (b c-a d)^2 \sqrt{f}\, (b e-a f)^2 \sqrt{\frac{e \left(c+d x^2\right)}{c \left(e+f x^2\right)}} \sqrt{e+f x^2}}$$

Result (type 4, 2744 leaves) :

$$\sqrt{c+d x^2} \sqrt{e+f x^2} \left(-\frac{d^3 x}{3 c (b c-a d) (-d e+c f)^2 \left(c+d x^2\right)^2}-\right.$$
$$\left.\frac{d^3 \left(-5 b c d e+2 a d^2 e+10 b c^2 f-7 a c d f\right) x}{3 c^2 (b c-a d)^2 (-d e+c f)^3 \left(c+d x^2\right)}+\frac{f^4 x}{e (b e-a f) (d e-c f)^3 \left(e+f x^2\right)}\right)+$$

$$\frac{1}{3 c^2 (b c-a d)^2 e (b e-a f) (-d e+c f)^3 \sqrt{c+d x^2} \sqrt{e+f x^2}}$$
$$\sqrt{\left(c+d x^2\right) \left(e+f x^2\right)} \left(\left(\left(5\, i\, b^2 c d^4 e^4 \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}}\right.\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]\right)\bigg/$$

$$\left(\sqrt{\frac{d}{c}}\;\sqrt{\left(c+d\,x^2\right)\left(e+f\,x^2\right)}\right)-\left(2\,i\,a\,b\,d^5\,e^4\,\sqrt{1+\frac{d\,x^2}{c}}\;\sqrt{1+\frac{f\,x^2}{e}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]\right)\bigg/$$

$$\left(\sqrt{\frac{d}{c}}\;\sqrt{\left(c+d\,x^2\right)\left(e+f\,x^2\right)}\right)-\left(10\,i\,b^2\,c^2\,d^3\,e^3\,f\,\sqrt{1+\frac{d\,x^2}{c}}\;\sqrt{1+\frac{f\,x^2}{e}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]\right)\bigg/$$

$$\left(\sqrt{\frac{d}{c}}\;\sqrt{\left(c+d\,x^2\right)\left(e+f\,x^2\right)}\right)+\left(2\,i\,a\,b\,c\,d^4\,e^3\,f\,\sqrt{1+\frac{d\,x^2}{c}}\;\sqrt{1+\frac{f\,x^2}{e}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]\right)\bigg/$$

$$\left(\sqrt{\frac{d}{c}}\;\sqrt{\left(c+d\,x^2\right)\left(e+f\,x^2\right)}\right)+\left(2\,i\,a^2\,d^5\,e^3\,f\,\sqrt{1+\frac{d\,x^2}{c}}\;\sqrt{1+\frac{f\,x^2}{e}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]\right)\bigg/$$

$$\left(\sqrt{\frac{d}{c}}\;\sqrt{\left(c+d\,x^2\right)\left(e+f\,x^2\right)}\right)+\left(10\,i\,a\,b\,c^2\,d^3\,e^2\,f^2\,\sqrt{1+\frac{d\,x^2}{c}}\;\sqrt{1+\frac{f\,x^2}{e}}\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]\right)\bigg/$$

$$\left(\sqrt{\frac{d}{c}}\;\sqrt{\left(c+d\,x^2\right)\left(e+f\,x^2\right)}\right)-\left(7\,i\,a^2\,c\,d^4\,e^2\,f^2\,\sqrt{1+\frac{d\,x^2}{c}}\;\sqrt{1+\frac{f\,x^2}{e}}\right.$$

$$\left(\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]-\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]\right)\right)\Big/$$

$$\left(\sqrt{\frac{d}{c}}\;\sqrt{\left(c+d\,x^2\right)\left(e+f\,x^2\right)}\right)-\left(3\,\text{i}\,b^2\,c^4\,d\,e\,f^3\sqrt{1+\frac{d\,x^2}{c}}\;\sqrt{1+\frac{f\,x^2}{e}}\right.$$

$$\left(\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]-\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]\right)\right)\Big/$$

$$\left(\sqrt{\frac{d}{c}}\;\sqrt{\left(c+d\,x^2\right)\left(e+f\,x^2\right)}\right)+\left(6\,\text{i}\,a\,b\,c^3\,d^2\,e\,f^3\sqrt{1+\frac{d\,x^2}{c}}\;\sqrt{1+\frac{f\,x^2}{e}}\right.$$

$$\left(\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]-\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]\right)\right)\Big/$$

$$\left(\sqrt{\frac{d}{c}}\;\sqrt{\left(c+d\,x^2\right)\left(e+f\,x^2\right)}\right)-\left(3\,\text{i}\,a^2\,c^2\,d^3\,e\,f^3\sqrt{1+\frac{d\,x^2}{c}}\;\sqrt{1+\frac{f\,x^2}{e}}\right.$$

$$\left(\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]-\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]\right)\right)\Big/$$

$$\left(\sqrt{\frac{d}{c}}\;\sqrt{\left(c+d\,x^2\right)\left(e+f\,x^2\right)}\right)+\left(4\,\text{i}\,b^2\,c^2\,d^3\,e^3\,f\sqrt{1+\frac{d\,x^2}{c}}\;\sqrt{1+\frac{f\,x^2}{e}}\right.$$

$$\left.\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]\right)\Big/\left(\sqrt{\frac{d}{c}}\;\sqrt{\left(c+d\,x^2\right)\left(e+f\,x^2\right)}\right)-$$

$$\left(\text{i}\,a\,b\,c\,d^4\,e^3\,f\sqrt{1+\frac{d\,x^2}{c}}\;\sqrt{1+\frac{f\,x^2}{e}}\;\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]\right)\Big/$$

$$\left(\sqrt{\frac{d}{c}}\;\sqrt{\left(c+d\,x^2\right)\left(e+f\,x^2\right)}\right)-$$

$$\left(9\,\text{i}\,b^2\,c^3\,d^2\,e^2\,f^2\sqrt{1+\frac{d\,x^2}{c}}\;\sqrt{1+\frac{f\,x^2}{e}}\;\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\;x\right],\frac{c\,f}{d\,e}\right]\right)\Big/$$

$$\left(\sqrt{\frac{d}{c}}\;\sqrt{\left(c+d\,x^2\right)\left(e+f\,x^2\right)}\right)+$$

$$\left(2\,\mathrm{i}\,a\,b\,c^2\,d^3\,e^2\,f^2\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)\Big/$$

$$\left(\sqrt{\frac{d}{c}}\,\sqrt{\left(c+d\,x^2\right)\,\left(e+f\,x^2\right)}\right)+$$

$$\left(\mathrm{i}\,a^2\,c\,d^4\,e^2\,f^2\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)\Big/$$

$$\left(\sqrt{\frac{d}{c}}\,\sqrt{\left(c+d\,x^2\right)\,\left(e+f\,x^2\right)}\right)-$$

$$\left(3\,\mathrm{i}\,b^2\,c^4\,d\,e\,f^3\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)\Big/$$

$$\left(\sqrt{\frac{d}{c}}\,\sqrt{\left(c+d\,x^2\right)\,\left(e+f\,x^2\right)}\right)+$$

$$\left(15\,\mathrm{i}\,a\,b\,c^3\,d^2\,e\,f^3\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)\Big/$$

$$\left(\sqrt{\frac{d}{c}}\,\sqrt{\left(c+d\,x^2\right)\,\left(e+f\,x^2\right)}\right)-$$

$$\left(9\,\mathrm{i}\,a^2\,c^2\,d^3\,e\,f^3\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)\Big/$$

$$\left(\sqrt{\frac{d}{c}}\,\sqrt{\left(c+d\,x^2\right)\,\left(e+f\,x^2\right)}\right)+$$

$$\left(3\,\mathrm{i}\,b^3\,c^2\,d^3\,e^4\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)\Big/$$

$$\left(a\,\sqrt{\frac{d}{c}}\,\sqrt{\left(c+d\,x^2\right)\,\left(e+f\,x^2\right)}\right)-$$

$$\left(9\,\mathrm{i}\,b^3\,c^3\,d^2\,e^3\,f\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)\Big/$$

$$\left(a\sqrt{\frac{d}{c}}\sqrt{\left(c+d x^2\right)\left(e+f x^2\right)}\right)+$$

$$\left(9\,i\,b^3 c^4 d\,e^2 f^2\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticPi}\left[\frac{b c}{a d},\,i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c f}{d e}\right]\right)\bigg/$$

$$\left(a\sqrt{\frac{d}{c}}\sqrt{\left(c+d x^2\right)\left(e+f x^2\right)}\right)-$$

$$\left(3\,i\,b^3 c^5 e\,f^3\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticPi}\left[\frac{b c}{a d},\,i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\,\frac{c f}{d e}\right]\right)\bigg/$$

$$\left(a\sqrt{\frac{d}{c}}\sqrt{\left(c+d x^2\right)\left(e+f x^2\right)}\right)\Bigg)$$

Problem 89: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(1+x^2\right)^{3/2}\sqrt{2+x^2}}{a+b x^2}\,dx$$

Optimal (type 4, 242 leaves, 7 steps):

$$-\frac{(a-2 b)\,x\sqrt{2+x^2}}{b^2\sqrt{1+x^2}}+\frac{x\sqrt{1+x^2}\sqrt{2+x^2}}{3 b}+$$

$$\frac{\sqrt{2}\,(a-2 b)\sqrt{2+x^2}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{b^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}-\frac{(3 a-7 b)\sqrt{2+x^2}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{3\sqrt{2}\,b^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}+$$

$$\frac{(a-2 b)(a-b)\sqrt{2+x^2}\,\text{EllipticPi}\left[1-\frac{b}{a},\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\,a\,b^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}$$

Result (type 4, 204 leaves):

$$\frac{1}{3\,a\,b^3}\left(a\,b^2\,x\,\sqrt{1+x^2}\,\sqrt{2+x^2}+3\,i\,a\,(a-2\,b)\,b\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-i\,a\,\left(3\,a^2-9\,a\,b+7\,b^2\right)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+3\,i\,a^3\,\text{EllipticPi}\left[\frac{2\,b}{a},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-12\,i\,a^2\,b\,\text{EllipticPi}\left[\frac{2\,b}{a},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+15\,i\,a\,b^2\,\text{EllipticPi}\left[\frac{2\,b}{a},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-6\,i\,b^3\,\text{EllipticPi}\left[\frac{2\,b}{a},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 90: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{1+x^2}\,\sqrt{2+x^2}}{a+b\,x^2}\,dx$$

Optimal (type 4, 192 leaves, 6 steps):

$$\frac{x\,\sqrt{2+x^2}}{b\,\sqrt{1+x^2}}-\frac{\sqrt{2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{b\,\sqrt{1+x^2}\,\sqrt{\frac{2+x^2}{1+x^2}}}+\frac{\sqrt{2+x^2}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\,b\,\sqrt{1+x^2}\,\sqrt{\frac{2+x^2}{1+x^2}}}-\frac{(a-2\,b)\,\sqrt{2+x^2}\,\text{EllipticPi}\left[1-\frac{b}{a},\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\,a\,b\,\sqrt{1+x^2}\,\sqrt{\frac{2+x^2}{1+x^2}}}$$

Result (type 4, 71 leaves):

$$\frac{1}{\sqrt{2}\,a\,b^2}i\left(-2\,a\,b\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],\frac{1}{2}\right]+(a-b)\,\left(a\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],\frac{1}{2}\right]-(a-2\,b)\,\text{EllipticPi}\left[\frac{b}{a},i\,\text{ArcSinh}[x],\frac{1}{2}\right]\right)\right)$$

Problem 91: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,\left(a+b\,x^2\right)}\,dx$$

Optimal (type 4, 58 leaves, 1 step):

$$\frac{2\,\sqrt{1+x^2}\,\text{EllipticPi}\left[1-\frac{2\,b}{a},\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],-1\right]}{a\,\sqrt{\frac{1+x^2}{2+x^2}}\,\sqrt{2+x^2}}$$

Result (type 4, 50 leaves):

$$-\frac{1}{\sqrt{2}\,a\,b}i\left(a\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],\frac{1}{2}\right]-(a-2\,b)\,\text{EllipticPi}\left[\frac{b}{a},i\,\text{ArcSinh}[x],\frac{1}{2}\right]\right)$$

Problem 92: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{2+x^2}}{\left(1+x^2\right)^{3/2}\left(a+b x^2\right)}\,dx$$

Optimal (type 4, 121 leaves, 3 steps):

$$\frac{\sqrt{2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{(a-b)\,\sqrt{1+x^2}\,\sqrt{\frac{2+x^2}{1+x^2}}}-\frac{2\,b\,\sqrt{1+x^2}\,\text{EllipticPi}\left[1-\frac{2\,b}{a},\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],-1\right]}{a\,(a-b)\,\sqrt{\frac{1+x^2}{2+x^2}}\,\sqrt{2+x^2}}$$

Result (type 4, 122 leaves):

$$\frac{1}{2\,a-2\,b}\left(\frac{2\,x\,\sqrt{2+x^2}}{\sqrt{1+x^2}}+2\,i\,\sqrt{2}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],\frac{1}{2}\right]-i\,\sqrt{2}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],\frac{1}{2}\right]-\right.$$
$$\left.i\,\sqrt{2}\,\text{EllipticPi}\left[\frac{b}{a},i\,\text{ArcSinh}[x],\frac{1}{2}\right]+\frac{2\,i\,\sqrt{2}\,b\,\text{EllipticPi}\left[\frac{b}{a},i\,\text{ArcSinh}[x],\frac{1}{2}\right]}{a}\right)$$

Problem 93: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{2+x^2}}{\left(1+x^2\right)^{5/2}\left(a+b x^2\right)}\,dx$$

Optimal (type 4, 215 leaves, 6 steps):

$$\frac{x\,\sqrt{2+x^2}}{3\,(a-b)\,\left(1+x^2\right)^{3/2}}+\frac{\sqrt{2}\,(a-2\,b)\,\sqrt{2+x^2}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{(a-b)^2\,\sqrt{1+x^2}\,\sqrt{\frac{2+x^2}{1+x^2}}}-$$
$$\frac{\sqrt{2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{3\,(a-b)\,\sqrt{1+x^2}\,\sqrt{\frac{2+x^2}{1+x^2}}}+\frac{2\,b^2\,\sqrt{1+x^2}\,\text{EllipticPi}\left[1-\frac{2\,b}{a},\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],-1\right]}{a\,(a-b)^2\,\sqrt{\frac{1+x^2}{2+x^2}}\,\sqrt{2+x^2}}$$

Result (type 4, 357 leaves):

$$\frac{1}{6\,a\,(a-b)^2\,(1+x^2)^2}\Big(8\,a^2\,x\,\sqrt{1+x^2}\,\sqrt{2+x^2}-14\,a\,b\,x\,\sqrt{1+x^2}\,\sqrt{2+x^2}+6\,a^2\,x^3\,\sqrt{1+x^2}\,\sqrt{2+x^2}-12\,a\,b\,x^3\,\sqrt{1+x^2}\,\sqrt{2+x^2}+6\,\mathrm{i}\,\sqrt{2}\,a\,(a-2\,b)\,(1+x^2)^2\,\text{EllipticE}\Big[\mathrm{i}\,\text{ArcSinh}[x],\frac{1}{2}\Big]-\mathrm{i}\,\sqrt{2}\,a\,(4\,a-7\,b)\,(1+x^2)^2\,\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}[x],\frac{1}{2}\Big]+3\,\mathrm{i}\,\sqrt{2}\,a\,b\,\text{EllipticPi}\Big[\frac{b}{a},\mathrm{i}\,\text{ArcSinh}[x],\frac{1}{2}\Big]-6\,\mathrm{i}\,\sqrt{2}\,b^2\,\text{EllipticPi}\Big[\frac{b}{a},\mathrm{i}\,\text{ArcSinh}[x],\frac{1}{2}\Big]+6\,\mathrm{i}\,\sqrt{2}\,a\,b\,x^2\,\text{EllipticPi}\Big[\frac{b}{a},\mathrm{i}\,\text{ArcSinh}[x],\frac{1}{2}\Big]-12\,\mathrm{i}\,\sqrt{2}\,b^2\,x^2\,\text{EllipticPi}\Big[\frac{b}{a},\mathrm{i}\,\text{ArcSinh}[x],\frac{1}{2}\Big]+3\,\mathrm{i}\,\sqrt{2}\,a\,b\,x^4\,\text{EllipticPi}\Big[\frac{b}{a},\mathrm{i}\,\text{ArcSinh}[x],\frac{1}{2}\Big]-6\,\mathrm{i}\,\sqrt{2}\,b^2\,x^4\,\text{EllipticPi}\Big[\frac{b}{a},\mathrm{i}\,\text{ArcSinh}[x],\frac{1}{2}\Big]\Big)$$

Problem 94: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{2+d\,x^2}\,\sqrt{3+f\,x^2}}{a+b\,x^2}\,dx$$

Optimal (type 4, 298 leaves, 6 steps):

$$\frac{f\,x\,\sqrt{2+d\,x^2}}{b\,\sqrt{3+f\,x^2}}-\frac{\sqrt{2}\,\sqrt{f}\,\sqrt{2+d\,x^2}\,\text{EllipticE}\Big[\text{ArcTan}\Big[\frac{\sqrt{f}\,x}{\sqrt{3}}\Big],1-\frac{3\,d}{2\,f}\Big]}{b\,\sqrt{\frac{2+d\,x^2}{3+f\,x^2}}\,\sqrt{3+f\,x^2}}+\frac{3\,d\,\sqrt{2+d\,x^2}\,\text{EllipticF}\Big[\text{ArcTan}\Big[\frac{\sqrt{f}\,x}{\sqrt{3}}\Big],1-\frac{3\,d}{2\,f}\Big]}{\sqrt{2}\,b\,\sqrt{f}\,\sqrt{\frac{2+d\,x^2}{3+f\,x^2}}\,\sqrt{3+f\,x^2}}+\frac{3\,(2\,b-a\,d)\,\sqrt{2+d\,x^2}\,\text{EllipticPi}\Big[1-\frac{3\,b}{a\,f},\text{ArcTan}\Big[\frac{\sqrt{f}\,x}{\sqrt{3}}\Big],1-\frac{3\,d}{2\,f}\Big]}{\sqrt{2}\,a\,b\,\sqrt{f}\,\sqrt{\frac{2+d\,x^2}{3+f\,x^2}}\,\sqrt{3+f\,x^2}}$$

Result (type 4, 134 leaves):

$$\frac{1}{\sqrt{3}\,a\,b^2\,\sqrt{d}}\mathrm{i}\,\Big(-3\,a\,b\,d\,\text{EllipticE}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{d}\,x}{\sqrt{2}}\Big],\frac{2\,f}{3\,d}\Big]+(-2\,b+a\,d)\,\Big(a\,f\,\text{EllipticF}\Big[\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{d}\,x}{\sqrt{2}}\Big],\frac{2\,f}{3\,d}\Big]+(3\,b-a\,f)\,\text{EllipticPi}\Big[\frac{2\,b}{a\,d},\mathrm{i}\,\text{ArcSinh}\Big[\frac{\sqrt{d}\,x}{\sqrt{2}}\Big],\frac{2\,f}{3\,d}\Big]\Big)\Big)$$

Problem 95: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{2+d\,x^2}}{\left(a+b\,x^2\right)\sqrt{3+f\,x^2}}\,dx$$

Optimal (type 4, 93 leaves, 1 step):

$$\frac{2\sqrt{3+f\,x^2}\;\text{EllipticPi}\left[1-\frac{2\,b}{a\,d},\,\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{2}}\right],\,1-\frac{2\,f}{3\,d}\right]}{\sqrt{3}\,a\,\sqrt{d}\,\sqrt{2+d\,x^2}\,\sqrt{\frac{3+f\,x^2}{2+d\,x^2}}}$$

Result (type 4, 94 leaves):

$$-\frac{1}{\sqrt{3}\,a\,b\,\sqrt{d}}\,i\left(a\,d\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{d}\,x}{\sqrt{2}}\right],\,\frac{2\,f}{3\,d}\right]+\left(2\,b-a\,d\right)\,\text{EllipticPi}\left[\frac{2\,b}{a\,d},\,i\,\text{ArcSinh}\left[\frac{\sqrt{d}\,x}{\sqrt{2}}\right],\,\frac{2\,f}{3\,d}\right]\right)$$

Problem 96: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+b\,x^2\right)\sqrt{2+d\,x^2}\,\sqrt{3+f\,x^2}}\,dx$$

Optimal (type 4, 49 leaves, 1 step):

$$\frac{\text{EllipticPi}\left[\frac{2\,b}{a\,d},\,\text{ArcSin}\left[\frac{\sqrt{-d}\,x}{\sqrt{2}}\right],\,\frac{2\,f}{3\,d}\right]}{\sqrt{3}\,a\,\sqrt{-d}}$$

Result (type 4, 52 leaves):

$$-\frac{i\,\text{EllipticPi}\left[\frac{2\,b}{a\,d},\,i\,\text{ArcSinh}\left[\frac{\sqrt{d}\,x}{\sqrt{2}}\right],\,\frac{2\,f}{3\,d}\right]}{\sqrt{3}\,a\,\sqrt{d}}$$

Problem 99: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{c-d\,x^2}\,\sqrt{e+f\,x^2}}{\left(a+b\,x^2\right)^2}\,dx$$

Optimal (type 4, 359 leaves, 11 steps):

$$\frac{x\sqrt{c-d x^2}\sqrt{e+f x^2}}{2 a\left(a+b x^2\right)}+\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{d x^2}{c}}\sqrt{e+f x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],-\frac{c f}{d e}\right]}{2 a b\sqrt{c-d x^2}\sqrt{1+\frac{f x^2}{e}}}-$$

$$\left(\sqrt{c}\sqrt{d}\,(b e+a f)\sqrt{1-\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],-\frac{c f}{d e}\right]\right)\Big/$$

$$\left(2 a b^2\sqrt{c-d x^2}\sqrt{e+f x^2}\right)+$$

$$\left(\sqrt{c}\left(b^2 c e+a^2 d f\right)\sqrt{1-\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticPi}\left[-\frac{b c}{a d},\text{ArcSin}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],-\frac{c f}{d e}\right]\right)\Big/$$

$$\left(2 a^2 b^2\sqrt{d}\sqrt{c-d x^2}\sqrt{e+f x^2}\right)$$

Result (type 4, 422 leaves) :

$$\left(\frac{c e x}{a+b x^2}-\frac{d e x^3}{a+b x^2}+\frac{c f x^3}{a+b x^2}-\frac{d f x^5}{a+b x^2}+\frac{1}{b}\right.$$

$$i c\sqrt{-\frac{d}{c}}\,e\sqrt{1-\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{d}{c}}\,x\right],-\frac{c f}{d e}\right]-\frac{1}{b^2}$$

$$i c\sqrt{-\frac{d}{c}}\,(b e+a f)\sqrt{1-\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{d}{c}}\,x\right],-\frac{c f}{d e}\right]+$$

$$\frac{1}{a\left(-\frac{d}{c}\right)^{3/2}}i d e\sqrt{1-\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticPi}\left[-\frac{b c}{a d},i\,\text{ArcSinh}\left[\sqrt{-\frac{d}{c}}\,x\right],-\frac{c f}{d e}\right]+$$

$$\left.\frac{1}{b^2}i a c\sqrt{-\frac{d}{c}}\,f\sqrt{1-\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\,\text{EllipticPi}\left[-\frac{b c}{a d},i\,\text{ArcSinh}\left[\sqrt{-\frac{d}{c}}\,x\right],-\frac{c f}{d e}\right]\right)\Big/$$

$$\left(2 a\sqrt{c-d x^2}\sqrt{e+f x^2}\right)$$

Problem 100: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{c+d x^2}\sqrt{e+f x^2}}{\left(a+b x^2\right)^2}\,dx$$

Optimal (type 4, 381 leaves, 8 steps) :

$$-\frac{f\,x\,\sqrt{c+d\,x^2}}{2\,a\,b\,\sqrt{e+f\,x^2}}+\frac{x\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}{2\,a\,\left(a+b\,x^2\right)}+\frac{\sqrt{e}\,\sqrt{f}\,\sqrt{c+d\,x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]}{2\,a\,b\,\sqrt{\frac{e\,(c+d\,x^2)}{c\,(e+f\,x^2)}}\,\sqrt{e+f\,x^2}}+$$

$$\frac{d\,\sqrt{e}\,\sqrt{f}\,\sqrt{c+d\,x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]}{2\,b^2\,c\,\sqrt{\frac{e\,(c+d\,x^2)}{c\,(e+f\,x^2)}}\,\sqrt{e+f\,x^2}}+$$

$$\left(\sqrt{-c}\,\left(b^2\,c\,e-a^2\,d\,f\right)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},\text{ArcSin}\left[\frac{\sqrt{d}\,x}{\sqrt{-c}}\right],\frac{c\,f}{d\,e}\right]\right)\Big/$$

$$\left(2\,a^2\,b^2\,\sqrt{d}\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}\right)$$

Result (type 4, 401 leaves):

$$\left(\frac{c\,e\,x}{a+b\,x^2}+\frac{d\,e\,x^3}{a+b\,x^2}+\frac{c\,f\,x^3}{a+b\,x^2}+\frac{d\,f\,x^5}{a+b\,x^2}+\right.$$

$$\frac{i\,c\,\sqrt{\frac{d}{c}}\,e\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]}{b}-\frac{1}{b^2}$$

$$i\,c\,\sqrt{\frac{d}{c}}\,\left(b\,e+a\,f\right)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]-$$

$$\frac{i\,c\,e\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]}{a\,\sqrt{\frac{d}{c}}}+\frac{1}{b^2}\,i\,a\,c\,\sqrt{\frac{d}{c}}\,f\,\sqrt{1+\frac{d\,x^2}{c}}$$

$$\left.\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)\Big/\left(2\,a\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}\right)$$

Problem 101: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(a+b\,x^2\right)^2\,\sqrt{c-d\,x^2}\,\sqrt{e+f\,x^2}}\,dx$$

Optimal (type 4, 426 leaves, 11 steps):

$$\frac{b^2 x \sqrt{c-d x^2}\ \sqrt{e+f x^2}}{2 a (b c+a d) (b e-a f) (a+b x^2)}+$$

$$\frac{b \sqrt{c}\ \sqrt{d}\ \sqrt{1-\frac{d x^2}{c}}\ \sqrt{e+f x^2}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{d}\ x}{\sqrt{c}}\right],-\frac{c f}{d e}\right]}{2 a (b c+a d) (b e-a f) \sqrt{c-d x^2}\ \sqrt{1+\frac{f x^2}{e}}}-$$

$$\frac{\sqrt{c}\ \sqrt{d}\ \sqrt{1-\frac{d x^2}{c}}\ \sqrt{1+\frac{f x^2}{e}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{d}\ x}{\sqrt{c}}\right],-\frac{c f}{d e}\right]}{2 a (b c+a d) \sqrt{c-d x^2}\ \sqrt{e+f x^2}}+$$

$$\left(\sqrt{c}\ \left(b^2 c e-3 a^2 d f+a b (2 d e-2 c f)\right) \sqrt{1-\frac{d x^2}{c}}\ \sqrt{1+\frac{f x^2}{e}}\ \text{EllipticPi}\left[-\frac{b c}{a d},\right.\right.$$

$$\left.\left.\text{ArcSin}\left[\frac{\sqrt{d}\ x}{\sqrt{c}}\right],-\frac{c f}{d e}\right]\right) \Big/ \left(2 a^2 \sqrt{d}\ (b c+a d) (b e-a f) \sqrt{c-d x^2}\ \sqrt{e+f x^2}\right)$$

Result (type 4, 773 leaves):

$$-\frac{b^2 x \sqrt{c-d x^2}\ \sqrt{e+f x^2}}{2 a (b c+a d) (-b e+a f) (a+b x^2)}+$$

$$\frac{1}{2 a (b c+a d) (-b e+a f) \sqrt{c-d x^2}\ \sqrt{e+f x^2}} \sqrt{(c-d x^2) (e+f x^2)}$$

$$\left(\left(\left(i b d e \sqrt{1-\frac{d x^2}{c}}\ \sqrt{1+\frac{f x^2}{e}}\ \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{d}{c}}\ x\right],-\frac{c f}{d e}\right]-\text{EllipticF}\left[\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left. i \text{ArcSinh}\left[\sqrt{-\frac{d}{c}}\ x\right],-\frac{c f}{d e}\right]\right)\right) \Big/ \left(\sqrt{-\frac{d}{c}}\ \sqrt{(c-d x^2) (e+f x^2)}\right)+$$

$$\left(i a d f \sqrt{1-\frac{d x^2}{c}}\ \sqrt{1+\frac{f x^2}{e}}\ \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{d}{c}}\ x\right],-\frac{c f}{d e}\right]\right) \Big/$$

$$\left(\sqrt{-\frac{d}{c}}\ \sqrt{(c-d x^2) (e+f x^2)}\right)+$$

$$\left(i b^2 c e \sqrt{1-\frac{d x^2}{c}}\ \sqrt{1+\frac{f x^2}{e}}\ \text{EllipticPi}\left[-\frac{b c}{a d}, i \text{ArcSinh}\left[\sqrt{-\frac{d}{c}}\ x\right],-\frac{c f}{d e}\right]\right) \Big/$$

$$\left(a\sqrt{-\frac{d}{c}}\sqrt{\left(c-d\,x^2\right)\left(e+f\,x^2\right)}\right)+$$

$$\left(2\,\mathrm{i}\,b\,d\,e\sqrt{1-\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[-\frac{b\,c}{a\,d},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{d}{c}}\,x\right],\,-\frac{c\,f}{d\,e}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{d}{c}}\sqrt{\left(c-d\,x^2\right)\left(e+f\,x^2\right)}\right)-$$

$$\left(2\,\mathrm{i}\,b\,c\,f\sqrt{1-\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[-\frac{b\,c}{a\,d},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{d}{c}}\,x\right],\,-\frac{c\,f}{d\,e}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{d}{c}}\sqrt{\left(c-d\,x^2\right)\left(e+f\,x^2\right)}\right)-$$

$$\left(3\,\mathrm{i}\,a\,d\,f\sqrt{1-\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[-\frac{b\,c}{a\,d},\,\mathrm{i}\,\text{ArcSinh}\left[\sqrt{-\frac{d}{c}}\,x\right],\,-\frac{c\,f}{d\,e}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{d}{c}}\sqrt{\left(c-d\,x^2\right)\left(e+f\,x^2\right)}\right)\Bigg)$$

Problem 102: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(a+b\,x^2\right)^2\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}\,dx$$

Optimal (type 4, 485 leaves, 8 steps):

$$-\frac{b\,f\,x\,\sqrt{c+d\,x^2}}{2\,a\,(b\,c-a\,d)\,(b\,e-a\,f)\,\sqrt{e+f\,x^2}}+\frac{b^2\,x\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}{2\,a\,(b\,c-a\,d)\,(b\,e-a\,f)\,(a+b\,x^2)}+$$

$$\frac{b\,\sqrt{e}\,\sqrt{f}\,\sqrt{c+d\,x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]}{2\,a\,(b\,c-a\,d)\,(b\,e-a\,f)\,\sqrt{\frac{e\,(c+d\,x^2)}{c\,(e+f\,x^2)}}\,\sqrt{e+f\,x^2}}-$$

$$\frac{d\,\sqrt{e}\,\sqrt{f}\,\sqrt{c+d\,x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{f}\,x}{\sqrt{e}}\right],1-\frac{d\,e}{c\,f}\right]}{2\,c\,(b\,c-a\,d)\,(b\,e-a\,f)\,\sqrt{\frac{e\,(c+d\,x^2)}{c\,(e+f\,x^2)}}\,\sqrt{e+f\,x^2}}+$$

$$\left(\sqrt{-c}\,\left(b^2\,c\,e+3\,a^2\,d\,f-2\,a\,b\,(d\,e+c\,f)\right)\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{b\,c}{a\,d},\text{ArcSin}\left[\frac{\sqrt{d}\,x}{\sqrt{-c}}\right],\frac{c\,f}{d\,e}\right]\right)\Big/\left(2\,a^2\,\sqrt{d}\,(b\,c-a\,d)\,(b\,e-a\,f)\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}\right)$$

Result (type 4, 587 leaves) :

$$\frac{1}{2\,a\,(-b\,c+a\,d)\,(-b\,e+a\,f)\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}\left(\frac{b^2\,c\,e\,x}{a+b\,x^2}+\frac{b^2\,d\,e\,x^3}{a+b\,x^2}+\frac{b^2\,c\,f\,x^3}{a+b\,x^2}+\right.$$

$$\frac{b^2\,d\,f\,x^5}{a+b\,x^2}+i\,b\,c\,\sqrt{\frac{d}{c}}\,e\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]-$$

$$i\,c\,\sqrt{\frac{d}{c}}\,(b\,e-a\,f)\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]-$$

$$\frac{1}{a\,\sqrt{\frac{d}{c}}}\,i\,b^2\,c\,e\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]+$$

$$2\,i\,b\,c\,\sqrt{\frac{d}{c}}\,e\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]+$$

$$\frac{1}{\sqrt{\frac{d}{c}}}\,2\,i\,b\,c\,f\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]-$$

$$\left.3\,i\,a\,c\,\sqrt{\frac{d}{c}}\,f\,\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\text{EllipticPi}\left[\frac{b\,c}{a\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{d}{c}}\,x\right],\frac{c\,f}{d\,e}\right]\right)$$

Problem 104: Unable to integrate problem.

$$\int\frac{\sqrt{a+b\,x^2}\,\sqrt{c+d\,x^2}}{\sqrt{e+f\,x^2}}\,dx$$

Optimal (type 4, 545 leaves, 7 steps):

$$\frac{d\,x\,\sqrt{a+b\,x^2}\,\sqrt{e+f\,x^2}}{2\,f\,\sqrt{c+d\,x^2}} -$$

$$\left(\sqrt{e}\,\sqrt{d\,e-c\,f}\,\sqrt{a+b\,x^2}\,\sqrt{\frac{c\,(e+f\,x^2)}{e\,(c+d\,x^2)}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{d\,e-c\,f}\,x}{\sqrt{e}\,\sqrt{c+d\,x^2}}\right],-\frac{(b\,c-a\,d)\,e}{a\,(d\,e-c\,f)}\right]\right)\Big/$$

$$\left(2\,f\,\sqrt{\frac{c\,(a+b\,x^2)}{a\,(c+d\,x^2)}}\,\sqrt{e+f\,x^2}\right)+$$

$$\left(b\,\sqrt{e}\,(d\,e-c\,f)\,\sqrt{c+d\,x^2}\,\sqrt{\frac{a\,(e+f\,x^2)}{e\,(a+b\,x^2)}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,e-a\,f}\,x}{\sqrt{e}\,\sqrt{a+b\,x^2}}\right],\frac{(b\,c-a\,d)\,e}{c\,(b\,e-a\,f)}\right]\right)\Big/$$

$$\left(2\,d\,f\,\sqrt{b\,e-a\,f}\,\sqrt{\frac{a\,(c+d\,x^2)}{c\,(a+b\,x^2)}}\,\sqrt{e+f\,x^2}\right)-$$

$$\left(c\,\sqrt{e}\,(b\,d\,e-b\,c\,f-a\,d\,f)\,\sqrt{a+b\,x^2}\,\sqrt{\frac{c\,(e+f\,x^2)}{e\,(c+d\,x^2)}}\,\text{EllipticPi}\left[\frac{d\,e}{d\,e-c\,f},\right.\right.$$

$$\left.\left.\text{ArcSin}\left[\frac{\sqrt{d\,e-c\,f}\,x}{\sqrt{e}\,\sqrt{c+d\,x^2}}\right],-\frac{(b\,c-a\,d)\,e}{a\,(d\,e-c\,f)}\right]\right)\Big/\left(2\,a\,d\,f\,\sqrt{d\,e-c\,f}\,\sqrt{\frac{c\,(a+b\,x^2)}{a\,(c+d\,x^2)}}\,\sqrt{e+f\,x^2}\right)$$

Result (type 8, 36 leaves) :

$$\int\frac{\sqrt{a+b\,x^2}\,\sqrt{c+d\,x^2}}{\sqrt{e+f\,x^2}}\,dx$$

Problem 105: Unable to integrate problem.

$$\int\frac{\sqrt{c+d\,x^2}}{\sqrt{a+b\,x^2}\,\sqrt{e+f\,x^2}}\,dx$$

Optimal (type 4, 163 leaves, 2 steps) :

$$\left(c\,\sqrt{e}\,\sqrt{a+b\,x^2}\,\sqrt{\frac{c\,(e+f\,x^2)}{e\,(c+d\,x^2)}}\,\text{EllipticPi}\left[\frac{d\,e}{d\,e-c\,f},\text{ArcSin}\left[\frac{\sqrt{d\,e-c\,f}\,x}{\sqrt{e}\,\sqrt{c+d\,x^2}}\right],-\frac{(b\,c-a\,d)\,e}{a\,(d\,e-c\,f)}\right]\right)\Big/$$

$$\left(a\,\sqrt{d\,e-c\,f}\,\sqrt{\frac{c\,(a+b\,x^2)}{a\,(c+d\,x^2)}}\,\sqrt{e+f\,x^2}\right)$$

Result (type 8, 36 leaves) :

$$\int\frac{\sqrt{c+d\,x^2}}{\sqrt{a+b\,x^2}\,\sqrt{e+f\,x^2}}\,dx$$

Problem 106: Unable to integrate problem.

$$\int \frac{\sqrt{c+d x^2}}{\left(a+b x^2\right)^{3/2} \sqrt{e+f x^2}} \, dx$$

Optimal (type 4, 148 leaves, 2 steps):

$$\frac{\sqrt{e} \sqrt{c+d x^2} \sqrt{\frac{a (e+f x^2)}{e (a+b x^2)}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b e-a f} x}{\sqrt{e} \sqrt{a+b x^2}}\right], \frac{(b c-a d) e}{c (b e-a f)}\right]}{a \sqrt{b e-a f} \sqrt{\frac{a (c+d x^2)}{c (a+b x^2)}} \sqrt{e+f x^2}}$$

Result (type 8, 36 leaves):

$$\int \frac{\sqrt{c+d x^2}}{\left(a+b x^2\right)^{3/2} \sqrt{e+f x^2}} \, dx$$

Problem 108: Unable to integrate problem.

$$\int \frac{\sqrt{a+b x^2} \sqrt{c+d x^2}}{\left(e+f x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 484 leaves, 8 steps):

$$-\frac{(d e-c f) x \sqrt{a+b x^2}}{e f \sqrt{c+d x^2} \sqrt{e+f x^2}}+\frac{\sqrt{c} \sqrt{d e-c f} \sqrt{a+b x^2} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d e-c f} x}{\sqrt{c} \sqrt{e+f x^2}}\right], -\frac{(b c-a d) e}{a (d e-c f)}\right]}{e f \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}} \sqrt{c+d x^2}}-$$

$$\frac{c^{3/2} (b e-a f) \sqrt{a+b x^2} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d e-c f} x}{\sqrt{c} \sqrt{e+f x^2}}\right], -\frac{(b c-a d) e}{a (d e-c f)}\right]}{a e f \sqrt{d e-c f} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}} \sqrt{c+d x^2}}+$$

$$\left(b c \sqrt{e} \sqrt{a+b x^2} \sqrt{\frac{c (e+f x^2)}{e (c+d x^2)}} \text{EllipticPi}\left[\frac{d e}{d e-c f},\right.\right.$$

$$\left.\left.\text{ArcSin}\left[\frac{\sqrt{d e-c f} x}{\sqrt{e} \sqrt{c+d x^2}}\right], -\frac{(b c-a d) e}{a (d e-c f)}\right]\right) \Big/ \left(a f \sqrt{d e-c f} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}} \sqrt{e+f x^2}\right)$$

Result (type 8, 36 leaves):

$$\int \frac{\sqrt{a+b x^2} \sqrt{c+d x^2}}{\left(e+f x^2\right)^{3/2}} \, dx$$

Problem 109: Unable to integrate problem.

$$\int \frac{\sqrt{c+d\,x^2}}{\sqrt{a+b\,x^2}\,\left(e+f\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 319 leaves, 5 steps):

$$\frac{\left(d\,e-c\,f\right)\,x\,\sqrt{a+b\,x^2}}{e\,\left(b\,e-a\,f\right)\,\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}-\frac{\sqrt{c}\,\sqrt{d\,e-c\,f}\,\sqrt{a+b\,x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d\,e-c\,f}\,x}{\sqrt{c}\,\sqrt{e+f\,x^2}}\right],-\frac{(b\,c-a\,d)\,e}{a\,(d\,e-c\,f)}\right]}{e\,\left(b\,e-a\,f\right)\,\sqrt{\frac{c\,\left(a+b\,x^2\right)}{a\,\left(c+d\,x^2\right)}}\,\sqrt{c+d\,x^2}}+\frac{c^{3/2}\,\sqrt{a+b\,x^2}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d\,e-c\,f}\,x}{\sqrt{c}\,\sqrt{e+f\,x^2}}\right],-\frac{(b\,c-a\,d)\,e}{a\,(d\,e-c\,f)}\right]}{a\,e\,\sqrt{d\,e-c\,f}\,\sqrt{\frac{c\,\left(a+b\,x^2\right)}{a\,\left(c+d\,x^2\right)}}\,\sqrt{c+d\,x^2}}$$

Result (type 8, 36 leaves):

$$\int \frac{\sqrt{c+d\,x^2}}{\sqrt{a+b\,x^2}\,\left(e+f\,x^2\right)^{3/2}}\,dx$$

Problem 111: Unable to integrate problem.

$$\int \frac{\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}{\sqrt{a+b\,x^2}}\,dx$$

Optimal (type 4, 541 leaves, 7 steps):

$$\frac{x\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}{2\sqrt{a+b\,x^2}}-\left(\sqrt{c}\,\sqrt{b\,c-a\,d}\,\sqrt{\frac{a\,(c+d\,x^2)}{c\,(a+b\,x^2)}}\,\sqrt{e+f\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b\,c-a\,d}\,x}{\sqrt{c}\,\sqrt{a+b\,x^2}}\right],\frac{c\,(b\,e-a\,f)}{(b\,c-a\,d)\,e}\right]\right)\Big/$$
$$\left(2\,b\sqrt{c+d\,x^2}\,\sqrt{\frac{a\,(e+f\,x^2)}{e\,(a+b\,x^2)}}\right)+\left((b\,c-a\,d)\,\sqrt{e}\,(2\,b\,e-a\,f)\,\sqrt{c+d\,x^2}\right.$$
$$\left.\sqrt{\frac{a\,(e+f\,x^2)}{e\,(a+b\,x^2)}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b\,e-a\,f}\,x}{\sqrt{e}\,\sqrt{a+b\,x^2}}\right],\frac{(b\,c-a\,d)\,e}{c\,(b\,e-a\,f)}\right]\right)\Big/$$
$$\left(2\,b^2\,c\,\sqrt{b\,e-a\,f}\,\sqrt{\frac{a\,(c+d\,x^2)}{c\,(a+b\,x^2)}}\,\sqrt{e+f\,x^2}\right)-\left(a\,(a\,d\,f-b\,(d\,e+c\,f))\,\sqrt{c+d\,x^2}\right.$$
$$\left.\sqrt{\frac{a\,(e+f\,x^2)}{e\,(a+b\,x^2)}}\,\text{EllipticPi}\left[\frac{b\,c}{b\,c-a\,d},\text{ArcSin}\left[\frac{\sqrt{b\,c-a\,d}\,x}{\sqrt{c}\,\sqrt{a+b\,x^2}}\right],\frac{c\,(b\,e-a\,f)}{(b\,c-a\,d)\,e}\right]\right)\Big/$$
$$\left(2\,b^2\,\sqrt{c}\,\sqrt{b\,c-a\,d}\,\sqrt{\frac{a\,(c+d\,x^2)}{c\,(a+b\,x^2)}}\,\sqrt{e+f\,x^2}\right)$$

Result (type 8, 36 leaves):

$$\int\frac{\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}{\sqrt{a+b\,x^2}}\,dx$$

Problem 113: Unable to integrate problem.

$$\int\frac{\sqrt{a+b\,x^2}}{\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}\,dx$$

Optimal (type 4, 159 leaves, 2 steps):

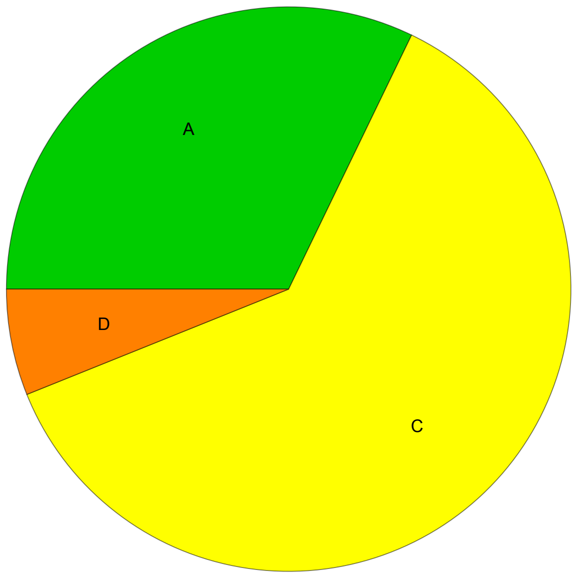
$$\left(a\sqrt{c+d\,x^2}\,\sqrt{\frac{a\,(e+f\,x^2)}{e\,(a+b\,x^2)}}\,\text{EllipticPi}\left[\frac{b\,c}{b\,c-a\,d},\text{ArcSin}\left[\frac{\sqrt{b\,c-a\,d}\,x}{\sqrt{c}\,\sqrt{a+b\,x^2}}\right],\frac{c\,(b\,e-a\,f)}{(b\,c-a\,d)\,e}\right]\right)\Big/$$
$$\left(\sqrt{c}\,\sqrt{b\,c-a\,d}\,\sqrt{\frac{a\,(c+d\,x^2)}{c\,(a+b\,x^2)}}\,\sqrt{e+f\,x^2}\right)$$

Result (type 8, 36 leaves):

$$\int\frac{\sqrt{a+b\,x^2}}{\sqrt{c+d\,x^2}\,\sqrt{e+f\,x^2}}\,dx$$

Summary of Integration Test Results

115 integration problems



A - 37 optimal antiderivatives

B - 0 more than twice size of optimal antiderivatives

C - 71 unnecessarily complex antiderivatives

D - 7 unable to integrate problems

E - 0 integration timeouts